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ANALYTICAL SOLUTION FOR MATHEMATICAL MODEL FOR COVID-19 DISEASE TRANSMISSION DYNAMICS INCORPORATING COMPARTMENT WITH NATURAL IMMUNITY USING HOMOTOPY PERTURBATION METHOD.

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ABSTRACT

In this current study, we presented a mathematical model for COVID-19 Disease transmission dynamics incorporating compartment with natural immunity. The model equations were solved for the state variables analytically via homotopy perturbation method (HPM). Numerical

Introduction

The coronavirus disease 2019, simply referred to as COVID-19, is a viral disease that was first reported in Wuhan China in 2019 (World health organisation, 2022). It is caused by Severe Acute Respiratory Syndrome coronavirus 2 (SARS-CoV-2). It has a high rate of infectivity but low mortality rate when compared with the outbreaks of the Middle East Respiratory Syndrome (MERS) 2012 and Severe Acute Respiratory Syndrome (SARS) 2003 (Imai et al. 2020 and Afshar et al. 2020). It is known to spread through exposure to the droplets from infected persons during coughing and sneezing. As the pandemic unfolds there are currently several vaccines in use, including AstraZeneca, Johnson and

simulations of the analytical results are carried out using computer symbolic algebraic package maple 17 and the graphical summaries of solutions were provided. Simulation results revealed that increase in the contact rate leads to increase in the individuals with natural immunity which in turn increases the spread of the disease. The solution shows also that Homotopy Perturbation Method (HPM) is an appropriate technique for solving the epidemic models.

Keywords: mathematical model, Coronavirus, Analytical Solution, Homotopy Perturbation Method.

Johnson, Modena, Pfizer and so on, approved by the world health organisation for the management of COVID-19. However, for many developing countries, getting the vaccines to administer to significant proportions of their populations has proven to be a tall order due to high demand among developed countries where these vaccines are manufactured. Consequently, in these developing countries efforts aimed at mitigating COVID-19 are mainly focusing on non-pharmaceutical interventions which include; using face masks, social-distancing, quarantine of suspected cases and contact tracing Isaac et al. (2021). These non-pharmaceutical containment measures proved to be a success in some countries, while the same measures failed in some countries probably due to non- adherence to the measures by the general populace, individual irresponsibility or inefficient contact tracing of asymptomatic and symptomatic cases Isaac et al. (2021). On the other hand, in developed countries where vaccines are available more than 60% of their population have received the first dose of the vaccine Jentsch et al. (2021). Many researchers work on models to investigate the activities of COVID-19 disease, see (Rahim, & Ehrahem 2020; Ambrosio &Aziz-Alaoui 2020; Wu, Leung & Leung 2020; Liang, K.2020; Zhou, Yang, Wang, Hu, Zhang, Si HR, Zhu, Huang, &Chen 2020), But their researched didn't include individual with natural immunity as a group. The homotopy perturbation method was introduced by the Chinese researcher Dr. Ji Huan HE in 1998 Tahmina & Mansur (2016). Recently this method became popular and acceptable as an elegant tool in the hands of researchers because of its simplicity and give rise highly effective solutions of complicated problems in many diverse areas of science and technology. Homotopy perturbation method is a straightforward and convenient method for both linear and nonlinear equations. This method does not depend on a small parameter. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$, which is considered as a small parameter Ahmet and Alev (2009). When $p = 0$, the system of equations usually reduces to a sufficiently simple form, which normally admits a rather simple solution. As p gradually increases to 1, the system goes through a sequence of deformations, the solution for each of which is close to that at the previous stage of deformation. Eventually at $p =1$, the system takes the original form of the equation and the final stage of deformation gives the desired solution. One of the most remarkable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution. Considerable research works have been conducted recently in applying this method to a class of linear and non-linear equations (Ahmet and Alev, 2009; Yildirim and Özis, 2007). For more knowledge of using HPM to find the solution of differential equation reader can see (Omale and Gochhai 2018; Mechee and Al-juaifri 2018; Peter and Awoniran 2018 and He, J. H. 1999). In this the present work we will use HPM to find the solutions of the model equation

MATERIAL AND METHODS

Description and Formulation of Model

The model here consist of five classes: $P(t)$ is the compartment used for those that are protected against the disease over a period of time. Protected individuals are recruited into the population at a rate $\omega\alpha$ and the population decrease by natural death at a rate μ. $S(t)$ is used to represent the number of individuals that are prone to the disease at time t, Susceptible individuals are recruited into the population at the rate $(1-\omega)\alpha$ by birth or emigration and also from treated class by losing temporary immunity at the rate ϕ and from protected class by losing protection at the rate λ, the population decrease by natural death at rate μ and by infection following a contact with infected individuals at a rate γ. $I(t)$ Is the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible categories, the population decreased by natural death, disease induced death and treatment. $N_i(t)$ Denotes the number of individuals who have been infected but poses natural immunity against the disease. $T(t)$ Is the number of individuals who have been infected with the disease and are treated, the population increases when infected individual get treated and move into the compartment, the population decrease by natural death at a rate μ . In the model we assumed that once an individual is treated, gets recovered from the disease, and there is reinfection once an individual is treated.

The model description can be written in the system of differential equations below:

$$\frac{dP}{dt} = \omega\alpha - (\mu + \kappa)P \tag{1}$$

$$\frac{dS}{dt} = (1-\omega)\alpha + \kappa P + \eta N_i + \phi T - (\mu + \gamma)S \tag{2}$$

$$\frac{dI}{dt} = \gamma S - (\mu + \delta + \tau + \beta)I \tag{3}$$

$$\frac{dN_i}{dt} = \tau I - (\mu + \eta)N_i \tag{4}$$

$$\frac{dT}{dt} = \beta I - (\mu + \phi)T \tag{5}$$

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With initial condition

$$P(0)=P_0>0,\ S(0)=S_0>0,\ I(0)=I_0>0,\ N_i(0)=N_{i0}>0,\ T(0)=T_0>0$$

Here, $\gamma=\frac{\varepsilon\varphi(1-d)}{N}$ is the effective force of infection.

Where ε the transmission probability rate of COVID-19, φ is the contact rate of infection, d is the effective rate of protection against infection.

The total population is given as:

$$N=P(t)+S(t)+I(t)+N_i(t)+T(t) \tag{6}$$

Model parameter and variable description

Table1: Definition of variables

Variables	Description
P	Total number of protected individuals at time t
S	Total number of susceptible individuals at time t
I	Total number of infected individuals at time t
N_i	Total number of individuals with natural immunity against at time t
T	Total number of treated individuals at time t.

Table 2: Definition of parameters

Parameter	Description
$\omega\alpha$	Is the at rate which protected individuals are recruited into the population
μ	Is the natural death rate
η	Is the rate at which natural immune individuals join susceptible class
ϕ	Is the rate of losing temporary immunity after treatment
γ	Is the rate of infection
κ	Is the rate of losing protection from the protected class
δ	**Is the rate of disease** induced death

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τ	Is the rate of natural recovery due to immunity after infection
β	Is the rate of treatment
φ	Is the contact rate of infection
ε	Is the transmission probability rate of COVID-19

Analytical Solution for the Model Equations using Homotopy perturbation Method.

The Homotopy Perturbation Method (HPM) provides analytical approximate solution that can applied to various linear and nonlinear equations. The HPM is a series expansion method used in the solution of nonlinear partial differential equation Jiya (2010). To demonstrate the concept of the homotopy perturbation method (HPM), the following differential equation was considered by He (2005):

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{6}$$

Subject to the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r = \Gamma \tag{7}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytical function, Γ is the boundary of the domain Ω and ∂ ∂n represents differentiation along the normal vector drawn outwards from Ω. The operator A can be divided into two parts of L and N, where L is the linear part and N is nonlinear. The operator A can be divided into two parts of L and N, where L is the linear part and N is the nonlinear component. Therefore, Equation (6) can be rewritten as:

$$L(u) + N(u) - f(r) = 0, \quad r \in \Omega \tag{8}$$

The structures of homotopy perturbation can b shown as:

$$H(v, F) = (1 - F)[L(v) - L(u_0)] + F[A(v) - f(r)] = 0, \quad r \in \Omega \tag{9}$$

Where, $v(r, F) : \Omega \in [0,1] \rightarrow \Re$ satisfies (9)

In equation (8), $F \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of equation (8) that satisfied the boundary condition.

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From equation (9),

$H(v,0) = L(v) - L(u_0) = 0$

$H(v,1) = L(v) + N(v) - f(r) = 0.$

The changing process of F from zero to unity is just that of $H(v,F)$ from $L(v) - L(u_0) = 0$ to $L(v) + N(v) - f(r) = 0$ is called deformation in topology, $L(v) - L(u_0)$ and $L(v) + N(v) - f(r)$ are called homotopic. It is assumed that solution of (8) can be written as power series in F as follows:

$$v = \sum_{i=0}^{\infty} v_i F^i \qquad (10)$$

As approaches 1, equation (9) reduces to (8) and the approximate solution to the problem is

$$\lim_{F \to 1} \sum_{i=0}^{\infty} v_i F^i \qquad (11)$$

The series (11) converges for most cases. However, the convergence rate depends on the nonlinear part of operator A(v).

Solution of the Model Equations

Equation (1) to (5) can be written as:

$$\frac{dP}{dt} + (\mu + \lambda)P - \omega\alpha = 0 \qquad (12)$$

$$\frac{dS}{dt} + (\mu + \gamma)S - (1 - \omega)\alpha - \lambda P - \eta N_i - \phi T = 0 \qquad (13)$$

$$\frac{dI}{dt} + (\mu + \delta + \tau + \beta)I - \gamma S \qquad (14)$$

$$\frac{dN_i}{dt} + (\mu + \eta)N_i - \tau I = 0 \qquad (15)$$

$$\frac{dT}{dt} + (\mu + \phi)T - \beta I = 0 \qquad (16)$$

With initial condition

$P(0)=P_0$
$S(0)=S_0$
$I(0)=I_0$
$N_i(0)=N_{i0}$
$T(0)=T_0$

Applying HPM on (12), we have

$$(1-F)\frac{dP}{dt}+F\left[\frac{dP}{dt}+(\mu+\lambda)P-\omega\alpha\right]=0 \quad (17)$$

Let

$$P=\sum_{i=0}^{\infty}F^i x_i \quad (18)$$

$$S=\sum_{i=0}^{\infty}F^i y_i \quad (19)$$

$$I=\sum_{i=0}^{\infty}F^i z_i \quad (20)$$

$$N_i=\sum_{i=0}^{\infty}F^i w_i \quad (21)$$

$$T=\sum_{i=0}^{\infty}q_i F^i \quad (22)$$

Substituting (18), (19), (20), (21) and (22) into equations (12) to (16) and expanding, we have

$$\begin{pmatrix} x_0+Fx_1+F^2x_2+\ldots+F\mu x_0+F^2\mu x_1+F^3\mu x_2+\ldots+ \\ F\lambda x_0+F^2\lambda x_1+F^3\lambda x_2+\ldots-F\omega\alpha=0 \end{pmatrix} \quad (23)$$

$$\begin{pmatrix} y_0+Fy_1+F^2y_2+\ldots+F\mu y_0+F^2\mu y_1+F^3\mu y_2+\ldots+F\gamma z_0y_0+F^2\gamma z_0y_1+ \\ F^3\gamma z_0y_2+\ldots+F^2\gamma z_1y_0+F^3\gamma z_1y_1+F^4\gamma z_1y_2+\ldots+F^3\gamma z_2y_0+F^4\gamma z_2y_1+ \\ F^5\gamma z_2y_2+\ldots-F\lambda x_0-F^2\lambda x_1-F^3\lambda x_2-F\eta w_0-F^2\eta w_1-F^3\eta w_2-F\phi q_0- \\ F^2\phi q_1-F^3\phi q_2-F(1-\omega)\alpha=0 \end{pmatrix}$$

(24)

$$\begin{pmatrix} z_0 + Fz_1 + F^2 z_2 + \ldots + F\mu z_0 + F^2 \mu z_1 + F^3 \mu z_2 + \ldots + F\delta z_0 + F^2 \delta z_1 + \\ F^3 \delta z_2 + \ldots + F\tau z_0 + F^2 \tau z_1 + F^3 \tau z_2 + \ldots F\beta z_0 + F^2 \beta z_1 + F^3 \beta z_2 - F\gamma z_0 y_0 - \\ F^2 \gamma z_0 y_1 - F^3 \gamma z_0 y_2 - \ldots - F^2 \gamma z_1 y_0 - F^3 \gamma z_1 y_1 - F^4 \gamma z_1 y_2 - \ldots - F^3 \gamma z_2 y_0 - \\ F^4 \gamma z_2 y_1 + F^5 \gamma z_2 y_2 - \ldots = 0 \end{pmatrix}$$

(25)

$$\begin{pmatrix} w_0 + Fw_1 + F^2 w_2 + \ldots + F\mu w_0 + F^2 \mu w_1 + F^3 \mu w_2 + \ldots + \\ F\mu w_0 + F^2 \eta w_1 + F^3 \eta w_2 + \ldots - F\tau z_0 - F^2 \tau z_1 - F^3 \tau z_2 - \ldots = 0 \end{pmatrix}$$

(26)

$$\begin{pmatrix} q_0 + Fq_1 + F^2 q_2 + \ldots + F\mu q_0 + F^2 \mu q_1 + F^3 \mu q_2 + \ldots + F\phi q_0 + \\ F^2 \phi q_1 + F^3 \phi q_2 + \ldots - F\beta z_0 - F^2 \beta z_1 - F^3 \beta z_2 - \ldots. = 0 \end{pmatrix}$$

(27)

Collecting the coefficients of powers of F form (23), we have:

$$F^0 : x^*{}_0 = 0 \quad (28)$$

$$F^1 : x^*{}_1 + \mu x_0 + \lambda x_0 - \omega\alpha = 0 \quad (29)$$

$$F^2 : x^*{}_2 + \mu x_1 + \lambda x_1 = 0 \quad (30)$$

$$F^3 : \mu x_2 + \mu x_1 + \lambda x_2 = 0 \quad (31)$$

Similarly from (24), we have

$$F^0 : y^*{}_0 = 0 \quad (32)$$

$$F^1 : y^*{}_1 + \mu y_0 + \gamma z_0 y_0 - \lambda x_0 - \eta w_0 - \phi q_0 - (1-\omega)\alpha = 0 \quad (33)$$

$$F^2 : y^*{}_2 + \mu y_1 + \gamma z_0 y_1 + \gamma z_1 y_0 - \lambda x_1 - \eta w_1 - \phi q_1 = 0 \quad (34)$$

$$F^3 : \mu y_2 + \gamma z_0 y_2 + \gamma z_1 y_1 + \gamma z_2 y_0 - \lambda x_2 - \eta w_2 - \phi q_2 = 0 \quad (35)$$

Similarly from (25)

$$F^0 : z^*{}_0 = 0 \quad (36)$$

$$F^1 : z^*{}_1 + \mu z_0 + \delta z_0 + \tau z_0 + \beta z_0 - \gamma z_0 y_0 = 0 \quad (37)$$

$$F^2 : z^*{}_2 + \mu z_1 + \delta z_1 + \tau z_1 + \beta z_0 - \gamma z_0 y_1 - \gamma z_1 y_0 = 0 \quad (38)$$

$$F^3 : \mu z_2 + \delta z_2 + \tau z_2 + \beta z_2 - \gamma z_0 y_2 - \gamma z_1 y_1 - \gamma z_2 y_0 = 0 \quad (39)$$

From (27), we have

$$F^0 : w^*{}_0 = 0 \quad (40)$$

$$F^1 : w^*_1 + \mu w_0 + \eta w_0 - \tau z_0 = 0 \quad (41)$$

$$F^2 : w^*_2 + \mu w_1 + \eta w_1 - \tau z_1 = 0 \quad (42)$$

$$F^3 : \mu w_2 + \eta w_2 - \tau z_2 = 0 \quad (43)$$

Similarly from (28)

$$F^0 : q^*_0 = 0 \quad (44)$$

$$F^1 : q^*_1 + \mu q_0 + \phi q_0 - \beta z_0 = 0 \quad (45)$$

$$F^2 : q^*_2 + \mu q_1 + \phi q_1 - \beta z_1 = 0 \quad (46)$$

$$F^3 : \mu q_2 + \phi q_2 - \beta z_2 = 0 \quad (47)$$

From equations (28), (32), (36), (40) and (44)

Integrating and applying initial conditions, we have

$$x_0 = P_0 \quad (48)$$

$$y_0 = S_0 \quad (49)$$

$$z_0 = I_0 \quad (50)$$

$$w_0 = N_{i0} \quad (51)$$

$$q_0 = T_0 \quad (52)$$

Solving equations (29), (33), (37), (41) and (45) by integrating and substituting (48) to (52) where necessary and applying initial conditions, we obtained

$$x_1 = (\omega\alpha - \mu P_0 - P_0)t \quad (53)$$

$$y_1 = [(1-\omega)\alpha + \lambda P_0 + \eta N_{i0} + \phi T_0 - \mu S_0 - \gamma I_0 S_0]t \quad (54)$$

$$z_1 = [(\gamma S_0 - \mu - \delta - \tau - \beta)I_0]t \quad (55)$$

$$w_1 = (\tau I_0 - \mu N_{i0} - \eta N_{i0})t \quad (56)$$

$$q_1 = (\beta I_0 - \mu q_0 - \phi q_0)t \quad (57)$$

From equation (30)

$$x_2 = -(\mu + \lambda)x_1 \quad (58)$$

Substituting (53) into (58), integrating and applying initial condition, we have

$$x_2 = -(\mu + \lambda)(\omega\alpha - \mu P_0 - \lambda P_0)\frac{t^2}{2} \quad (59)$$

As F $\to 1$, the solution of (18) can be obtained as:

$$P(t) = \lim_{F \to 1} \sum_{i=0}^{\infty} F^i x_i$$

Therefore,

$$P(t) = P_0 + (\omega\alpha - \mu P_0 - \lambda P_0)t - (\mu + \lambda)(\omega\alpha - \mu P_0 - \lambda P_0)\frac{t^2}{2} \quad (60)$$

Following same procedure from (58) to (60), we obtained the solutions of (19), (20), (21) and (22) as:

$$S(t) = S_0 + \begin{bmatrix} (1-\omega)\alpha + \lambda P_0 + \\ \eta N_{i0} + \phi T_0 - \mu S_0 \\ -\gamma I_0 S_0 \end{bmatrix} t + \begin{bmatrix} \lambda(\omega\alpha - \mu P_0 - \lambda P_0) + \eta(\tau I_0 - \mu N_{i0} - \eta N_{i0}) + \\ \phi(\beta I_0 - \mu q_0 - \phi q_0) - \gamma S_0(\gamma S_0 - \mu - \delta - \tau - \beta)I_0 \\ -(\mu + \gamma I_0)[(1-\omega)\alpha + \lambda P_0 + \eta N_{i0} + \phi T_0 - \mu S_0 - \gamma I_0 S_0] \end{bmatrix} \frac{t^2}{2}$$

(61)

$$I(t) = I_0 + [(\gamma S_0 - \mu - \delta - \tau - \beta)I_0]t + \begin{bmatrix} \gamma I_0[(1-\omega)\alpha + \lambda P_0 + \eta N_{i0} + \phi T_0 - \mu S_0 - \gamma I_0 S_0] \\ -(\mu + \delta + \tau + \beta - \gamma S_0)[(\gamma S_0 - \mu - \delta - \tau - \beta)I_0] \end{bmatrix} \frac{t^2}{2}$$

(62)

$$N_i(t) = N_{i0} + (\tau I_0 - \mu N_{i0} - \eta N_{i0})t + \begin{bmatrix} \tau(\gamma S_0 - \mu - \delta - \tau - \beta)I_0 - \\ (\mu + \eta)(\tau I_0 - \mu N_{i0} - \eta N_{i0}) \end{bmatrix} \frac{t^2}{2}$$

(63)

$$T(t) = T_0 + (\beta I_0 - \mu q_0 - \phi q_0)t + \begin{bmatrix} \beta I_0(\gamma S_0 - \mu - \delta - \tau - \beta) - \\ (\mu + \phi)(\beta I_0 - \mu T_0 - \phi T_0) \end{bmatrix} \frac{t^2}{2}$$

(64)

Hence, equations (60) to (64) are the general solutions of the model equations.

NUMERICAL SIMULATION AND GRAPHICAL ILLUSTRATION OF MODEL

Here, we present the numerical simulation which demonstrates the analytical results for the model. The simulation was carried out using computer symbolic algebraic package MAPLE 17.

Table 3: values of state variables and source

Variables	Value	Source
P(t)	200000	assumed
S(t)	1781500	assumed
E(t)	10000	assumed
F(t)	3000	assumed
T(t)	5500	assumed

Table 4: values of Parameters and source

Parameters	Values	Source
ω	0.3	assumed
α	2.5	assumed
δ	0.044	Iboi et al., 2020
η	0.07004	calculated
ε	0.2	assumed
κ	0.2	assumed
μ	0.014	Akwafuo et al., 2017
ϕ	0.00229	calculated
d	0.6	assumed
φ	0.15	assumed
τ	0.09820	calculated
β	0.0714	calculated

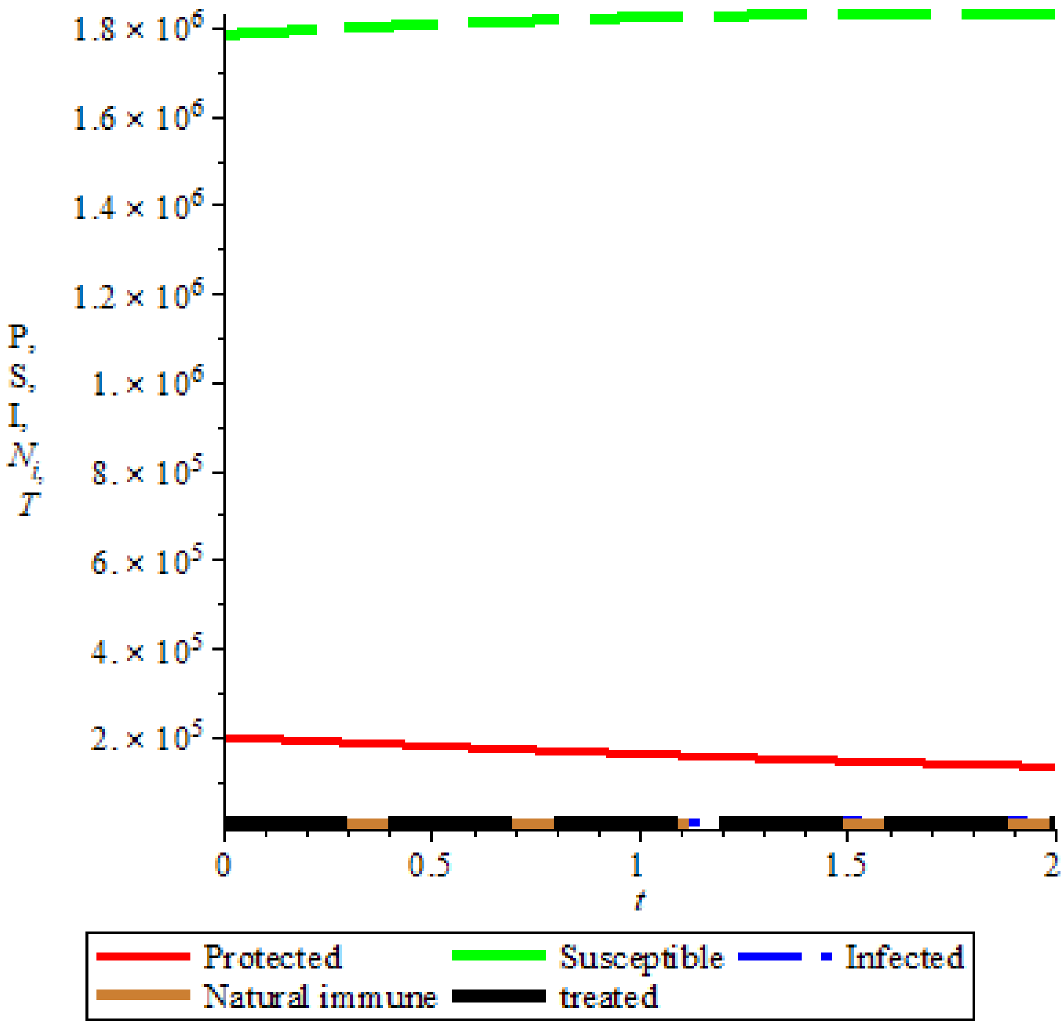

Figure1. Graph of state variables $P(t), S(t), I(t), N_i(t), T(t)$ – time (t) relationships at various values of recruitment rate $\alpha = 2, \alpha = 2.2, \alpha = 2.4, \alpha = 2.6, \alpha = 2.8$ and rate of losing protection $\kappa = 0.2, \kappa = 0.4, \kappa = 0.6, \kappa = 0.8, \kappa = 1$.

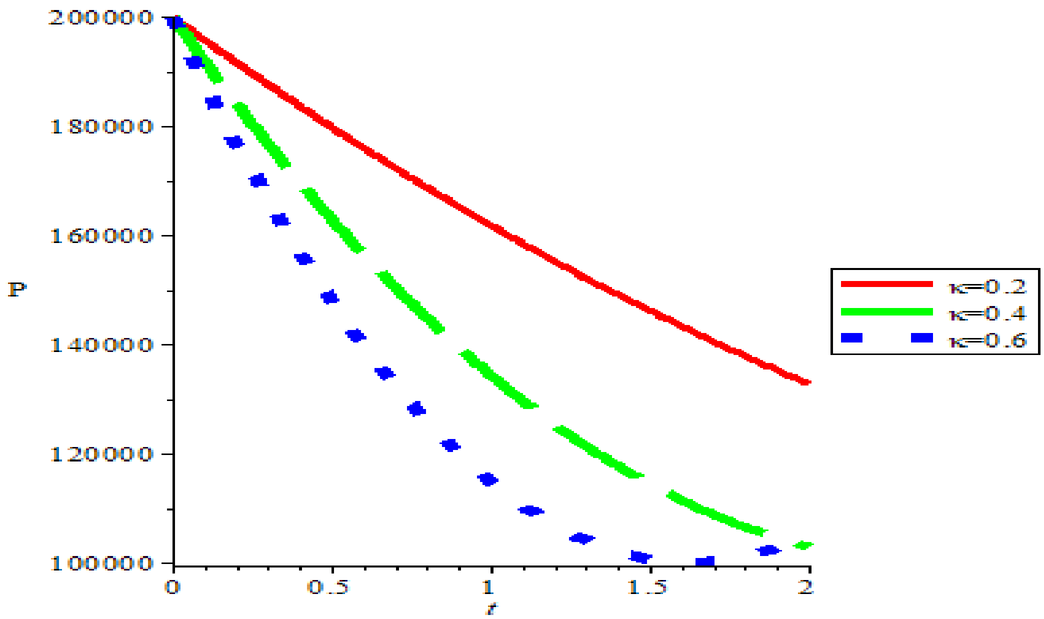

Figure 2: Protected individuals $P(t)$ – time (t) relationships at various values of rate of losing protection κ.

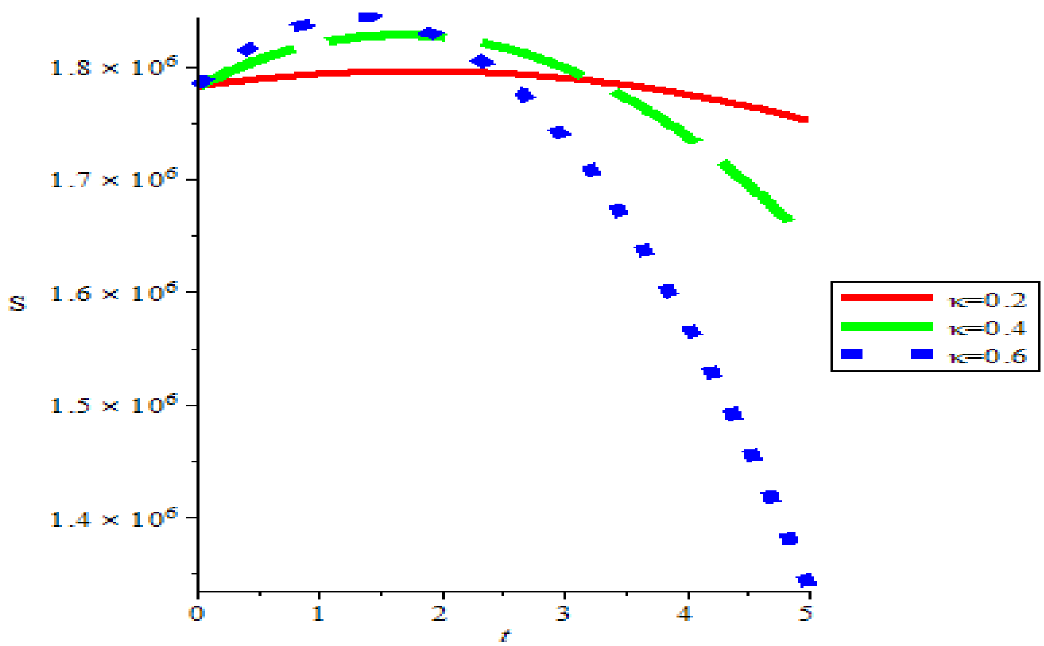

Figure 3: Susceptible individuals $S(t)$ – time (t) relationships at various values of rate of losing protection κ.

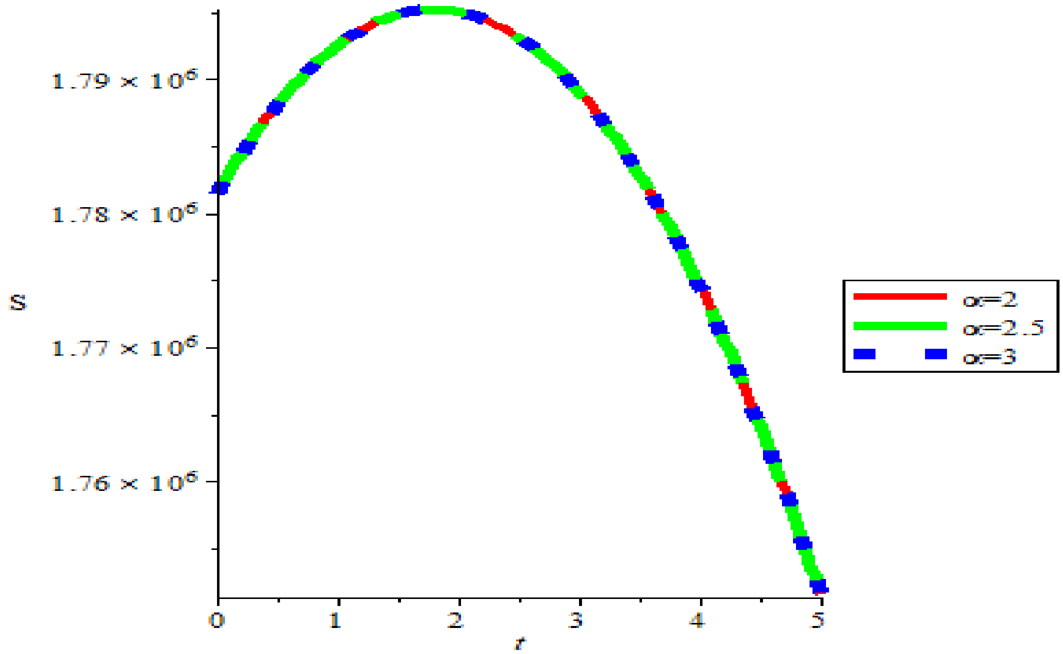

Figure 4: Susceptible individuals $S(t)$ – time (t) relationships at various values of recruitment rate α .

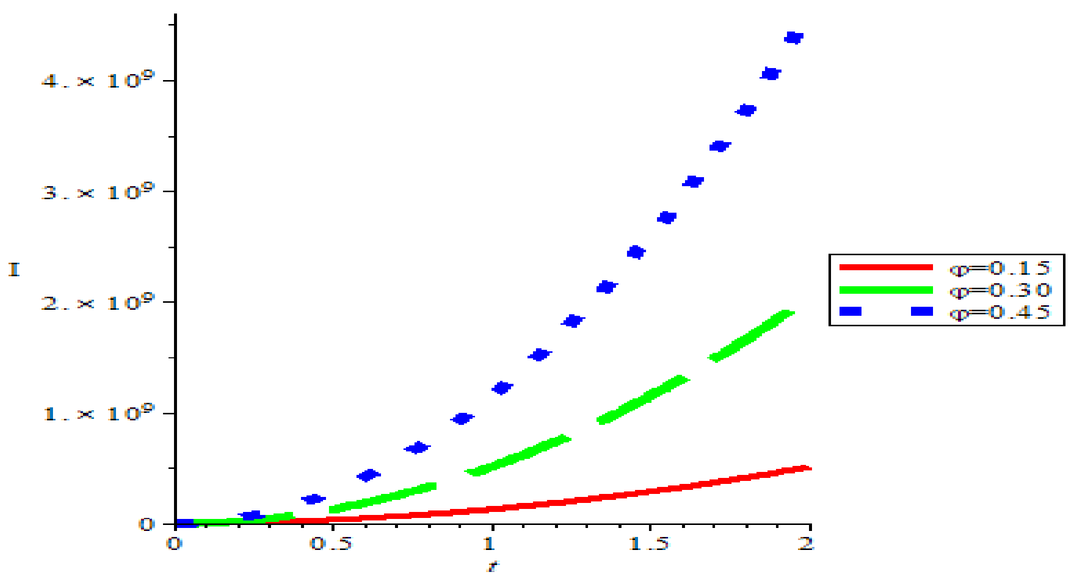

Figure 5: Infected individuals $I(t)$ – time (t) relationships at various values of contact rate φ .

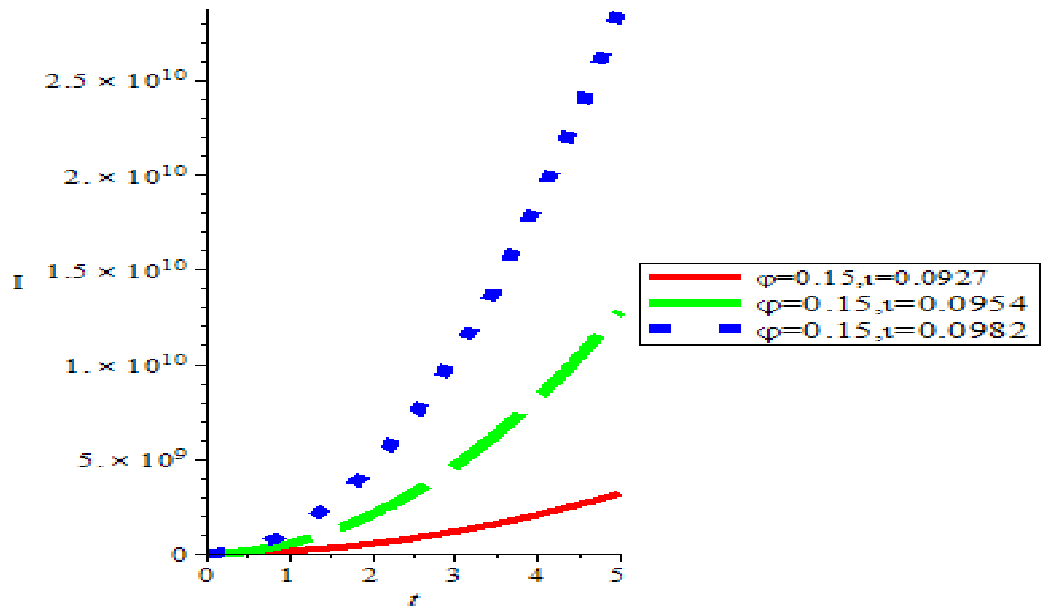

Figure 6: Infected individuals $I(t)$ – time (t) relationships at various values of contact rate φ and rate of acquiring natural immunity τ .

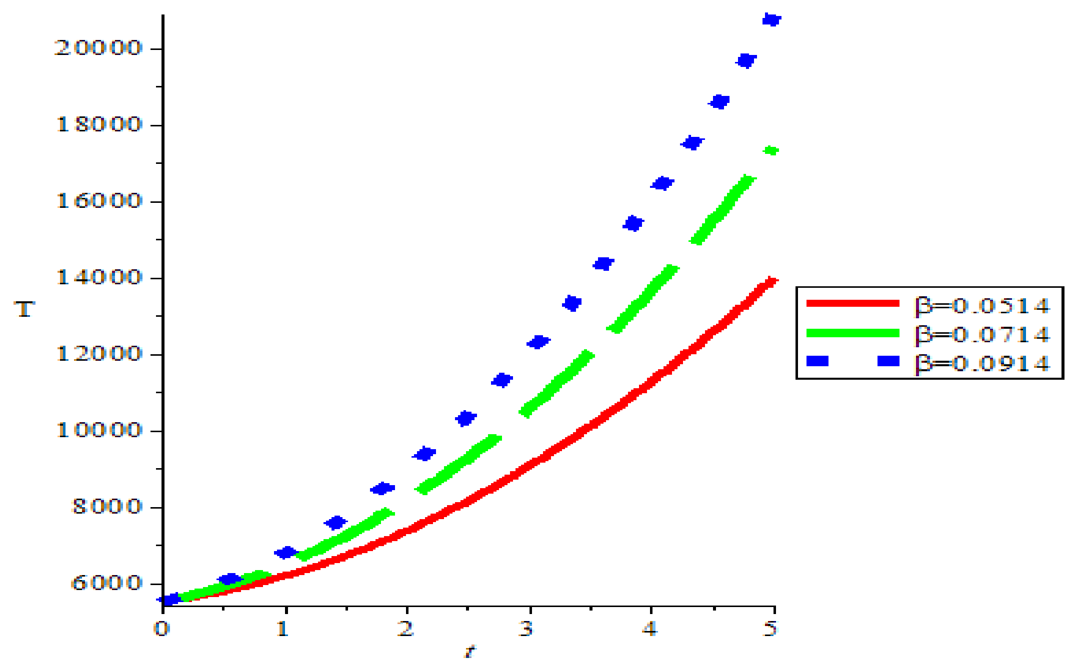

Figure 7: Treated individuals $T(t)$ – time (t) relationships at various values of rate of treatment β.

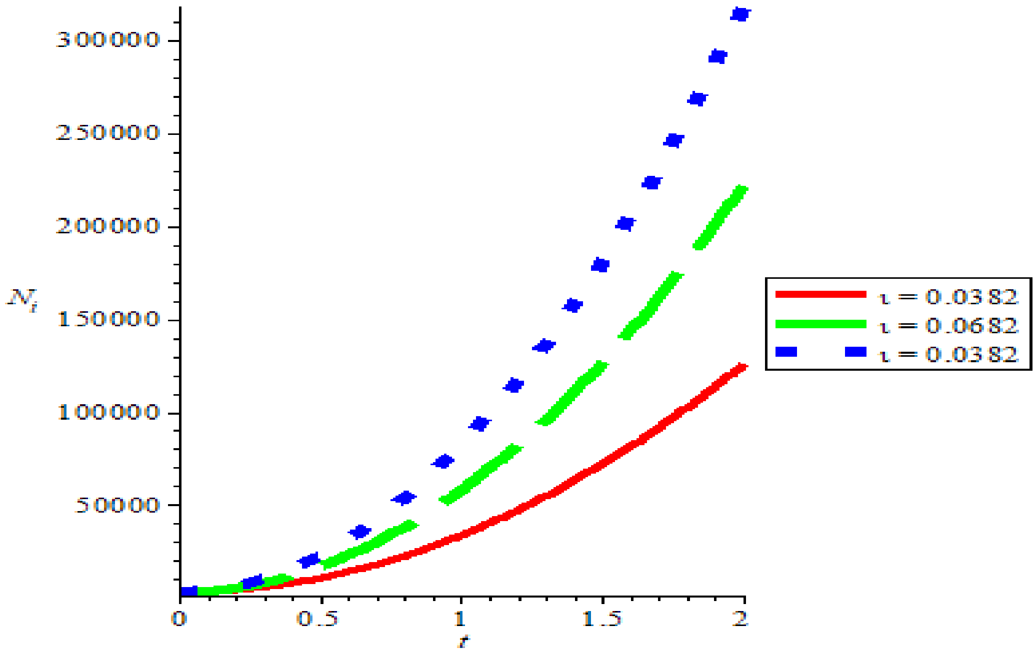

Figure 8: Individuals with natural immunity $N_i(t)$ – time (t) relationships at various values of rate of acquiring natural immunity τ.

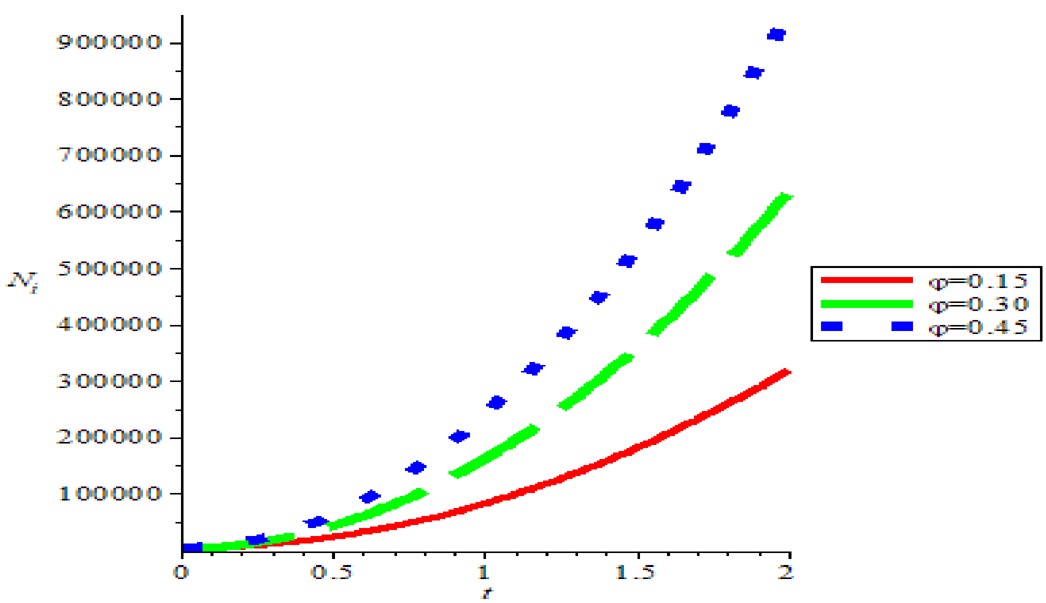

Figure 9: Individuals with natural immunity $N_i(t)$ – time (t) relationships at various values of contact rate φ

Discussion of Results

Figure1: shows the effect of recruitment rate and rate of losing protection on the state variables. It is observed that the Susceptible $S(t)$ individual increases with time t, but increases with increase in the recruitment rate and rate of losing protection which indicates direct relationship between susceptibility and recruitment rate and rate of losing protection. Also the protected individual decreases with time t, but decreases with increase in the rate of losing protection. It can also be observed that the infected $I(t)$, natural immunity and treated individuals curves do not raised up from the horizontal axis with time t, which indicates that contact rate is zero or not effective for the disease to spread in the population and in epidemiology, that is $N_i(t)=0$, $T(t)=0$ when $I(t)=0$.

Figure 2: shows the effect of rate of losing protection on the protected individuals. It is observed that the protected individuals decrease with time t, but decreases with increase in rate of losing protection κ. This implies that the higher the rate at which people losing protection, the lesser the protected individuals come.

Figure 3: shows the effect of rate of losing protection on the susceptible individuals. It is observed that the susceptible individual increases and later decreases with time t, but decreases with increase in the rate of losing protection.

Figure 4: shows the effect of recruitment rate on the susceptible individuals. It is observed that the susceptible individual increases and later decreases with time t, but decreases with increase in the recruitment rate.

Figure 5: shows the effect of contact rate on the infected individuals $I(t)$. It is observed that the infected individuals $I(t)$ increase with time t, but increases with increase in the contact rate. The graph shows that increase in the contact rate increases the infection. Figure 6: Shows the effect of contact rate φ and rate of acquiring natural immunity τ on infected individuals $I(t)$. It is observed that the infected individuals $I(t)$ increase with

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time t, but increases with increase in contact rate φ and rate of acquiring natural immunity τ . This implies that increase in contact rate and rate of acquiring natural immunity lead to increase in the infection. Figure 7: shows the effect of treatment rate on the treated individuals $T(t)$. It is observed that the treated individuals $T(t)$ increase with time t, but increases with increase in the of rate treatment. The graph shows that increase in the rate of treatment increases the treated population. Figure 8: shows the effect of rate of acquiring natural immunity on the individual with natural immunity. It is observed that the natural immune individual increases with time t, but increases with increase in the rate of acquiring natural immunity. This means that the population with natural immunity increases due to increase in the rate of acquiring natural immunity. Figure 9: shows the effect of contact rate on the individual with natural immunity. It is observed that the natural immune individual increase with time t, but increases with increase in the contact. The graph shows that the higher the contact rate, the more people acquire natural immunity.

Conclusion

We have formulated and solved analytically a mathematical model for COVID-19 Disease transmission dynamics incorporating compartment with natural immunity using Homotopy Perturbation Method (HPM). Numerical simulations of the analytical results are carried out using computer symbolic algebraic package maple 17 and the graphical summaries of solutions were provided. The solution shows that Homotopy Perturbation Method (HPM) is an appropriate technique for solving the epidemic models.

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