



# QUALITATIVE BEHAVIOUR OF BIOCHEMICAL-OXYGEN DEMAND AND DISSOLVED-OXYGEN INTERACTIONS FOR A MILD ENVIRONMENTAL PERTURBATION ON DODO RIVER.

## ABSTRACT

In studying the qualitative behaviour of biochemical-oxygen demand (BOD) and dissolved-oxygen (DO) interactions for a mild environmental perturbation on the BOD, we have used a computational method of the Runge-Kutta ODE45 numerical scheme for the analysis. The results outputs shows that due to a slight change in the initial condition of the inclusion of a mild random environmental perturbation value of  $RNI=0.1$  on the BOD coordinate for  $DO(IC=4.13)$  and  $BOD(IC=2.08)$  portrays a threat to the aquatic environment as the grey areas indicates a depletion in the coordinates of the

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## Introduction

The qualitative behaviour of biochemical-oxygen demand (BOD) and dissolved-oxygen (DO) interactions for a mild environmental perturbation on the BOD is a challenging environmental problem that will be solved computationally using numerical simulation approach (Akpodee & Ekaka-a, 2019) on an interacting DO and BOD mathematical model formulation on a polluted river for water quality and aquatic species survival. Sewage disposal, municipal dumps, artisanal refineries, effluent discharge and several other unchecked activities aid in polluting our water bodies thereby making the aquatic environment vulnerable to BOD as the toxic pollutants outweighs the DO meant for the usage and survival of the aquatic species - a situation that can lead to the biological extinction of the aquatic species within the highly river. Research has shown that the river of high-profile cities like Port-Harcourt, Warri and Yenagoa in Nigeria are highly polluted due the regular discharge of toxic effluent water. In this study, pollution will be analysed at the highly polluted study area and a dynamical mathematical model will be use to solve this complex environmental problem. This work is intended to summarise the harmful effects of water pollution and the threat to aquatic species in order to provide a solution using mathematical modelling that will useful in predicting the changes in water quality and stop further damage. In this research work, it has been closely observed that the water bodies become polluted due to the release of untreated waste, sewage, dissolved oxygen, bacteria and toxic chemical substances from industries which is worsening the physical, chemical and biological properties of water. The mathematical



DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a critical  $DO_{mc} = 2.0295$  against the actual  $DO_c = 2.6357$  at the 21<sup>st</sup> day and recovery at  $DO_m = 1.8670$  against actual recovery  $DO = 2.6677$  on the 26<sup>th</sup> day of our experimental time before the control. In the same scenario, due to the inclusion of a mild environmental perturbation on the BOD coordinates for a random noise intensity value of 0.1, we observed a slight gain in the coordinate of the BOD with  $BOD_o (IC = 2.08)$  on the base day which depleted to  $BOD_m = 1.3714$  against the actual  $BOD = 1.5171$  on the Sixth (6<sup>th</sup>) day. The  $BOD_m$  coordinates due to the modification, fluctuates drastically from  $BOD_m = 1.3714$  on the sixth (6<sup>th</sup>) day to a converging (saturating) value of  $BOD_m = 0.4248$  on the 361<sup>st</sup> day of our experimental time. The detailed results of this study is fully presented and discussed accordingly.

**Keywords:** Random Noise Intensity, Environmental Perturbation Biochemical-Oxygen Demand, Dissolved Oxygen, Numerical Simulation, Biological Extinction, Model Parameters.

concepts used for one-dimensional habitat to solve non-linear differential equations will be applied to solve this complex problem.

#### MATHEMATICAL FORMULATION

When water of river is polluted then the river has an ability to purify itself using some chemical and biological actions. This is known as self-purification. Self-purification can be proved as a good indicator for the status of a river, whenever pollutants is there in a river; 2 process takes place simultaneously.

1. De-oxygen
2. Re-aeration

The interaction of these two processes gives the river its self-cleansing property.

The first-order decay equation of industrial wastes for the two processes is as follows (Kaushik, 2015):

$$\dot{X} = -DC^rY + KT_c^r(S - X) \quad (1)$$

$$\dot{Y} = -DC^rY \quad (2)$$

D = decay rate/day

K = proportional coefficient

C = correction coefficient

$\tau = T - 20$

X = Dissolved Oxygen (D.O); Y = Biochemical-Oxygen Demand (B.O.D).

$T_c$  = temperature correction coefficient

S = saturation concentration

#### Analytical Solution

$$\frac{dX}{dt} = -DC^rY + (S - X), \quad X(0) = X_0 > 0 \quad (3)$$

$$\frac{dY}{dt} = -DC^rY, \quad Y(0) = Y_0 > 0 \quad (4)$$



Solving equation (3.4) by separation of the variable

$$\frac{dY}{dt} = -DC^{\tau}Y$$

$$\frac{dY}{Y} = -DC^{\tau}dt$$

Integrating both sides,

$$\int \frac{1}{Y} dY = -DC^{\tau} \int dt$$

$$\log_e Y(t) = -DC^{\tau}t + h$$

Taking exponential of both side

$$Y(t) = e^{(-DC^{\tau}t+h)} = e^{-DC^{\tau}t} \cdot e^h = Ae^{-DC^{\tau}t} \quad \text{Where } A = e^h \quad (5)$$

$$y(0) = y_0$$

$$y(0) = y_0 = e^{-DC^{\tau}(0)} \cdot A = A$$

$$y_0 = A$$

Thus

$$y(t) = y_0 e^{-DC^{\tau}t} \quad (6)$$

Substituting equation (3.6) into equation (3.3)

$$\begin{aligned} \frac{dX}{dt} &= -DC^{\tau}y_0 e^{-DC^{\tau}t} + KT_c^{\tau}(S - X) \\ \frac{dX}{dt} &= -DC^{\tau}y_0 e^{-DC^{\tau}t} + KT_c^{\tau}S - XKT_c^{\tau} \\ \frac{dX}{dt} + XKT_c^{\tau} &= KT_c^{\tau}S - DC^{\tau}y_0 e^{-DC^{\tau}t} \end{aligned} \quad (7)$$

The equation is a linear inhomogeneous first order ODE with integrating factor (I.F) of the form

$$\frac{dy}{dx} + p(x)y = Q(x) \equiv \frac{dX}{dt} + p(t)X = Q(t) \quad (8)$$

where  $I.F = e^{\int p(x)dx} \equiv e^{\int p(t)dt}$

hence,  $I.F = e^{\int KT_c^{\tau}dt} = e^{KT_c^{\tau}t}$

multiplying I.F with equation (3.7) we have

$$\begin{aligned} e^{KT_c^{\tau}t} \frac{dX}{dt} + e^{KT_c^{\tau}t} XKT_c^{\tau} &= (KT_c^{\tau}S - DC^{\tau}y_0 e^{-DC^{\tau}t}) e^{KT_c^{\tau}t} \\ \frac{d}{dt} (X e^{KT_c^{\tau}t}) &= KT_c^{\tau}S e^{KT_c^{\tau}t} - DC^{\tau}y_0 e^{-DC^{\tau}t} \cdot e^{KT_c^{\tau}t} \\ \frac{d}{dt} (X e^{KT_c^{\tau}t}) &= KT_c^{\tau}S e^{KT_c^{\tau}t} - DC^{\tau}y_0 e^{-(DC^{\tau}-KT_c^{\tau})t} \end{aligned} \quad (9)$$

Separating variable

$$d(X e^{KT_c^{\tau}t}) = KT_c^{\tau}S e^{KT_c^{\tau}t} dt - DC^{\tau}y_0 e^{-(DC^{\tau}-KT_c^{\tau})t} dt$$

Integrating both side

$$\begin{aligned} \int \frac{d}{dt} (X e^{KT_c^{\tau}t}) &= KT_c^{\tau}S \int e^{KT_c^{\tau}t} dt - DC^{\tau}y_0 \int e^{-(DC^{\tau}-KT_c^{\tau})t} dt \\ X e^{KT_c^{\tau}t} &= \frac{KT_c^{\tau}S}{KT_c^{\tau}} e^{KT_c^{\tau}t} - \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})} e^{-(DC^{\tau}-KT_c^{\tau})t} + h_1 \end{aligned}$$

Dividing through by  $e^{KT_c^{\tau}t}$ , we have

$$\begin{aligned} X(t) &= \frac{KT_c^{\tau}S}{KT_c^{\tau}} + \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau}) e^{KT_c^{\tau}t}} \cdot e^{-(DC^{\tau}-KT_c^{\tau})t} + h_1 e^{-KT_c^{\tau}t} \\ X(t) &= S + \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})} \cdot e^{-(DC^{\tau})t} + h e^{-KT_c^{\tau}t} \\ X(0) &= X_0 \end{aligned}$$



$$X(0) = X_0 = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} + h$$

$$h = X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)}$$

$$X(t) = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \cdot e^{-(DC^\tau)t} + \left( X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \right) e^{-KT_c^\tau t} \quad (10)$$

Thus, the predictive model for dissolved oxygen (DO) and Bio-oxygen Demand (BOD) for arbitrary parameter values are stated as follows

$$X(t) = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \cdot e^{-(DC^\tau)t} + \left( X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \right) e^{-KT_c^\tau t}$$

$$Y(t) = y_0 e^{-DC^\tau t}$$

### METHOD OF SOLUTION

The numerical simulation aspects using the above solution trajectories of the BOD-DO model will be the core of this studies.

Following Akpodee (2019), when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows:

The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function  $f(x, y)$  in the initial value problem

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition  $y(x_0) = y_0$

The increment  $h$  in  $x$  may be changed at each step, but is usually kept constant so that after  $n$  steps, we have

$$x_n = x_0 + nh$$

The Runge-Kutta algorithm for the determination of the approximation  $y_{n+1}$  to  $y(x_{n+1})$  is

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_{n+1}, y_n + k_3)$$

The local error involved in the determination of  $y_{n+1}$  from  $y_n$  is  $O(h^5)$

The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z)$$



$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial condition  $y(x_0) = y_0$  and  $z(x_0) = z_0$

These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.

At the  $n$ th integration step, using a step of length  $h$ , the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

Where,

$$k_1 = hf(x_n, y_n, z_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2)$$

$$k_4 = hf(x_n + h, y_n + k_3, z_n + K_3)$$

and

$$K_1 = hg(x_n, y_n, z_n)$$

$$K_2 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1)$$

$$K_3 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2)$$

$$K_4 = hg(x_n + h, y_n + k_3, z_n + K_3)$$

As with the Runge-Kutta method, the local error involved in the determination of  $y_{n+1}$  from  $y_n$  and  $z_{n+1}$  from  $z_n$  is  $O(h^5)$

It is a good numerical analysis practice that in the event of a complex dynamical system that can not admit an analytic solution for sensitivity analysis of interacting variables, we have to adopt an alternative method to study the qualitative behavior of the unique positive co-existence steady-state solution of the DO and BOD as  $t \rightarrow \infty$ . This is a challenging environmental problem that will be tackle computationally using MATLAB ODE45 numerical scheme.

For the purpose of this study, we have used the following parameter values compressed from the field data stated as follows:

$d_1$  = BOD decay rate/day

$k$  = Oxygen Supply Proportional coefficient

$C$  = BOD correction coefficient

$\tau$  = Oxygen Supply Temperature Difference =  $T - 20$

$X$  = DO

$Y$  = BOD

$T$  = Temperature

$T_c$  = temperature correction coefficient for Oxygen Supply

$S$  = saturation concentration for DO

$T_0=20$ ;  $T=28.91$ ;  $\tau = T-T_0$ ;  $T_c=1.0300$ ;  $k=0.01$ ;  $d_1=0.1172$ ,  $X(0) = 4.13$ ,  $Y(0)=2.08$ ;  $S = 4.50$



**Results and Discussion**

Table 1.1: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 0(5)90 in days

Example	Time (Days)	DO(mg/l)	DO <sub>m</sub> (mg/l)	BOD (mg/l)	BOD <sub>m</sub> (mg/l)
1	0	4.1300	4.1300	2.0800	2.0800
2	5.0000	3.2625	3.1972	1.1571	1.3714
3	10.0000	2.8458	2.6205	0.6445	0.9853
4	15.0000	2.6743	2.2596	0.3588	0.7387
5	20.0000	2.6357	2.0295	0.1997	0.5856
6	25.0000	2.6677	1.8670	0.1112	0.5247
7	30.0000	2.7356	1.7496	0.0619	0.4827
8	35.0000	2.8203	1.6611	0.0345	0.4462
9	40.0000	2.9114	1.5933	0.0192	0.4358
10	45.0000	3.0032	1.5338	0.0107	0.4294
11	50.0000	3.0930	1.4744	0.0059	0.4362
12	55.0000	3.1790	1.4146	0.0033	0.4470
13	60.0000	3.2608	1.3671	0.0018	0.4313
14	65.0000	3.3381	1.3194	0.0010	0.4287
15	70.0000	3.4109	1.2789	0.0006	0.4232
16	75.0000	3.4792	1.2401	0.0003	0.4445
17	80.0000	3.5434	1.1942	0.0002	0.4344
18	85.0000	3.6036	1.1564	0.0001	0.4373
19	90.0000	3.6600	1.1179	0.0001	0.4228

Table 1.2: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 95(5)180 in days

Example	Time (Days)	DO (mg/l)	DO <sub>m</sub> (mg/l)	BOD (mg/l)	BOD <sub>m</sub> (mg/l)
1	95.0000	3.7129	1.0967	0.0000	0.4094
2	100.0000	3.7625	1.0757	0.0000	0.4208
3	105.0000	3.8089	1.0474	0.0000	0.4297
4	110.0000	3.8524	1.0194	0.0000	0.4380
5	115.0000	3.8932	0.9912	0.0000	0.4423
6	120.0000	3.9315	0.9633	0.0000	0.4537
7	125.0000	3.9673	0.9360	0.0000	0.4457
8	130.0000	4.0008	0.9168	0.0000	0.4224
9	135.0000	4.0323	0.9002	0.0000	0.4222
10	140.0000	4.0617	0.8907	0.0000	0.4210
11	145.0000	4.0893	0.8711	0.0000	0.4549



12	150.0000	4.1152	0.8445	0.0000	0.4431
13	155.0000	4.1395	0.8222	-0.0000	0.4446
14	160.0000	4.1622	0.8046	-0.0000	0.4359
15	165.0000	4.1834	0.7904	-0.0000	0.4345
16	170.0000	4.2034	0.7799	0.0000	0.4232
17	175.0000	4.2221	0.7755	0.0000	0.4204
18	180.0000	4.2396	0.7659	-0.0000	0.4191

**Table 1.3: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 185(5)270 in days**

Example	Time (Days)	DO (mg/l)	DO <sub>m</sub> (mg/l)	BOD (mg/l)	BOD <sub>m</sub> (mg/l)
1	185.0000	4.2560	0.7559	0.0000	0.4448
2	190.0000	4.2714	0.7336	0.0000	0.4709
3	195.0000	4.2858	0.7148	0.0000	0.4326
4	200.0000	4.2993	0.7103	0.0000	0.4343
5	205.0000	4.3119	0.6990	0.0000	0.4414
6	210.0000	4.3237	0.6920	0.0000	0.4424
7	215.0000	4.3349	0.6895	0.0000	0.4095
8	220.0000	4.3453	0.6856	0.0000	0.4368
9	225.0000	4.3550	0.6767	0.0000	0.4452
10	230.0000	4.3641	0.6670	0.0000	0.4418
11	235.0000	4.3727	0.6564	0.0000	0.4495
12	240.0000	4.3807	0.6519	0.0000	0.4250
13	245.0000	4.3882	0.6512	0.0000	0.4207
14	250.0000	4.3953	0.6572	0.0000	0.4170
15	255.0000	4.4019	0.6622	0.0000	0.4262
16	260.0000	4.4080	0.6606	0.0000	0.4434
17	265.0000	4.4138	0.6633	0.0000	0.4155
18	270.0000	4.4193	0.6640	0.0000	0.4400



Table 1.4: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of  $RNI=0.1$  on the BOD coordinate for  $DO(IC=4.13)$  and  $BOD(IC=2.08)$  interactions between a time interval of 275(5)360 in days

Example	Time (Days)	DO (mg/l)	DO <sub>m</sub> (mg/l)	BOD (mg/l)	BOD <sub>m</sub> (mg/l)
1	275.0000	4.4244	0.6562	0.0000	0.4337
2	280.0000	4.4291	0.6563	0.0000	0.4270
3	285.0000	4.4336	0.6487	0.0000	0.4393
4	290.0000	4.4378	0.6380	0.0000	0.4555
5	295.0000	4.4417	0.6315	0.0000	0.4348
6	300.0000	4.4454	0.6293	0.0000	0.4230
7	305.0000	4.4488	0.6334	0.0000	0.4300
8	310.0000	4.4520	0.6324	0.0000	0.4392
9	315.0000	4.4551	0.6270	0.0000	0.4426
10	320.0000	4.4579	0.6251	0.0000	0.4352
11	325.0000	4.4605	0.6312	0.0000	0.4019
12	330.0000	4.4630	0.6388	0.0000	0.4229
13	335.0000	4.4654	0.6388	0.0000	0.4284
14	340.0000	4.4675	0.6406	0.0000	0.4175
15	345.0000	4.4696	0.6454	0.0000	0.4193
16	350.0000	4.4715	0.6478	0.0000	0.4132
17	355.0000	4.4733	0.6447	0.0000	0.4382
18	360.0000	4.4750	0.6377	0.0000	0.4248

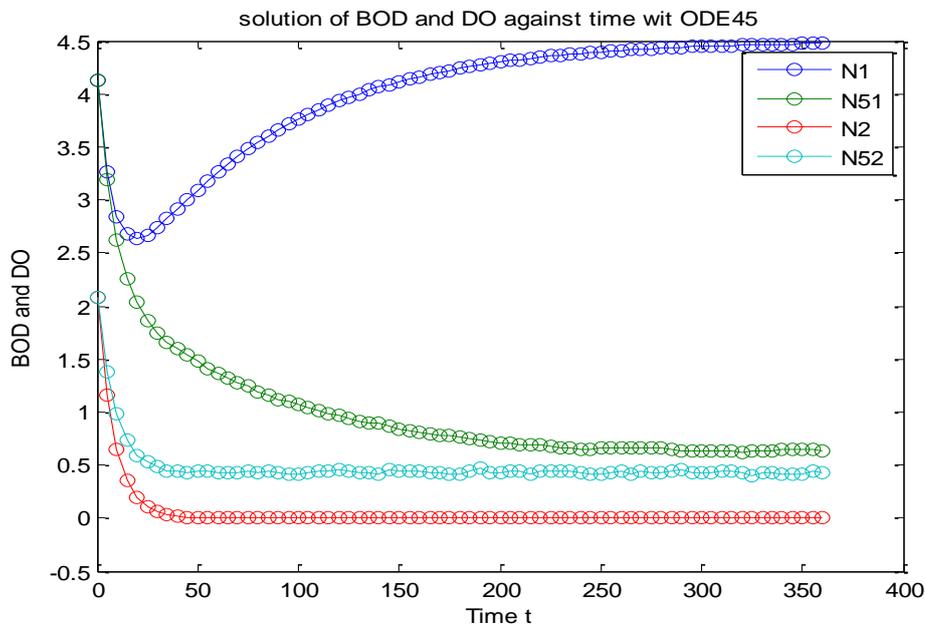


Figure 1.1: Solution trajectory of Scenario I of the impact of the inclusion of a mild random environmental perturbation value of  $RNI=0.1$  on the BOD coordinate for  $DO(IC=4.13)$  and  $BOD(IC=2.08)$  interactions between a time interval of 0(5)360 in days



From the result obtained in Table 1.1 – Table 1.4 and Figure 4.12, we observed that when all model parameter values are fixed ranging from the time interval of 0:5:360 in days, the BOD and DO initial values here called the initial condition (IC) on the base day are recorded as  $BOD_o = 2.080$  and  $DO_o = 4.130$  due to pollution of the stream. Furthermore, as a result of the impact of the inclusion of a medium random environmental perturbation value  $RNI = 0.1$  on the BOD coordinate for  $DO(IC=4.13)$  and  $BOD(IC=2.08)$  in their interaction between a time interval at 0(50)90 in days, we observed grey area such as the time interval between (0 – 15) days called the interval of degradation with  $DO_o = 4.13$  on the base day up  $DO_m = 2.2960$  on the sixteenth (16<sup>th</sup>) day against the actual the actual  $DO = 4.13$  on the base day up to  $DO = 2.6743$  on the sixteenth (16<sup>th</sup>) day before the control. The next grey area is the interval of active decomposition which is observed between (10 – 35) days with  $DO_m = 2.6205$  on the eleventh (11<sup>th</sup>) day and  $DO_m = 1.6611$  on the 36<sup>th</sup> day against the actual  $DO = 2.8458$  on the 11<sup>th</sup> day and  $DO = 2.8203$  on the 36<sup>th</sup> day before the control. The DO deficit of the modified coordinate is recorded as  $DO_{mc} = 2.0295$  at the critical time (days) which fall on the twenty first (21<sup>st</sup>) day of our experimental time. Moreover, a further grey area captured is the interval of active recovery which ranges from (35 – 150) days has  $DO_m = 1.1661$  on the 36<sup>th</sup> day and  $DO_m = 0.8445$  on the 151<sup>st</sup> day been the point of inflexion against the actual  $DO = 2.6205$  on the 36<sup>th</sup> day and  $DO = 1.6611$  on the 151<sup>st</sup> day been the actual value of the point of inflexion before the control. Here, an important region or grey area called “the interval of complete saturation” has  $DO_m = 0.8445$  at the point of inflexion on the 151<sup>st</sup> day and a saturated value of  $DO_m = 0.6377$  on the 361<sup>st</sup> day of our experimental time against the actual  $DO = 4.1152$  on the 151<sup>st</sup> day and  $DO = 4.4750$  on the 361<sup>st</sup> day before control.

This observation due to a slight change in the initial condition of the inclusion of a mild random environmental perturbation value of  $RNI=0.1$  on the BOD coordinate for  $DO(IC=4.13)$  and  $BOD(IC=2.08)$  portrays a threat to the aquatic environment as the grey areas indicates a depletion in the coordinates of the DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a critical  $DO_{mc} = 2.0295$  against the actual  $DO_c = 2.6357$  at the 21<sup>st</sup> day and recovery at  $DO_m = 1.8670$  against actual recovery  $DO = 2.6677$  on the 26<sup>th</sup> day of our experimental time before the control. In the same scenario, due to the inclusion of a mild environmental perturbation of random noise intensity value of 0.1 on the BOD coordinates, we observed a slight gain in the coordinate of the BOD with  $BOD_o (IC = 2.08)$  on the base day which depleted to  $BOD_m = 1.3714$  against the actual  $BOD = 1.5171$  on the Sixth (6<sup>th</sup>) day. The  $BOD_m$  coordinates due to the modification, decreases drastically from  $BOD_m = 1.3714$  on the sixth (6<sup>th</sup>) day to a converging (saturating) value of  $BOD_m = 0.4248$  on the 361<sup>st</sup> day of our experimental time.

This observation is consistent and vital for environmental decision and policy making which will improve the scope of protecting the water bodies and aquatic environment as a result of pollution. The information discussed here captures grey area of vulnerability of the aquatic environment and will serve as a valuable tool that may be utilized by environmental protection agency and other related bodies concerned with pollution for monitoring, intervention and mitigation.

## CONCLUSION

In studying the impact of environmental perturbation on polluted stream using analytical method and describing the qualitative characterization of the dynamical system will be quite tasking. Hence we have introduced the core method for this work which is the computational



method in order to get an early insight of the expected results and also eliminate approximation errors. Through this method, we were able to predict the relative abundance of the BOD-DO in their interactions over a period of 360 *at an interval* 0: 5: 360 for a mild random environmental perturbation with random noise intensity value of 0.1 using MatLab ODE45 numerical scheme. It was observed that there was a decrease in the relative abundance of both coordinates and a point of recovery in the DO coordinates with its DO deficit up to its complete saturation and a randomized value of the BOD coordinates.

The differential equation model for effluent discharge used in this work is very effective in the determination and prediction of the relative abundance of the BOD-DO coordinates over a period of time.

### RECOMMENDATION

To researchers who may want to do similar works as this, we recommend that:

The impact of a low and severe environmental perturbation should be studied.

The impact of the growth rate should be checked using computational method.

The impact varying initial conditions should be predicted using computational method.

Similar method should be adopted to study the qualitative behavior other physiochemical properties over time.

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