



ABSTRACT

This challenging problem was tackled computationally using MATLAB ODE45 Runge-Kutta numerical scheme for a time interval of 0(5)360 in days. due to a slight change in the initial condition of DO(IC = 4.0) and BOD (IC = 2.3) portrays a threat to the aquatic environment as the grey areas indicate a depletion in the coordinates of the DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a

ANALYSIS OF THE EFFECT OF CHANGING INITIAL CONDITION OF BIOCHEMICAL-OXYGEN DEMAND(IC=2.3) AND DISSOLVED-OXYGEN(IC=4.0) IN THEIR INTERACTIONS ON DODO RIVER.

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Introduction

Human activities especially industrialization is a major contributor to the degradation and pollution of the environment which has undesirably affected the water bodies (Owa, 2013). The discharge of effluent with high pollutant concentration into rivers in the Niger delta region of Nigeria as a result of oil spillage, industrial activities, sewage disposal and several other sources of water pollution in the area is of great concern as its impact on water quality and most importantly aquatic lives portends great threat to ecosystem functionality in the region. The justification that there is steady oil spill in the region which finds its way to the river due to water level convenience and the fact that most industries discharge their highly polluted toxic effluent without thorough treatment up to world health organization (WHO) standard into the water bank is the basic rational that underpins this study. Sewage disposal, municipal dumps, artisanal refineries, effluent discharge and several other unchecked activities aid in polluting the water body thereby making the aquatic environment vulnerable to Biochemical Oxygen Demand (BOD) as the toxic pollutants outweighs the Dissolved Oxygen (DO) meant for the usage and survival of the aquatic species - a situation that can lead to the biological extinction of the



critical $DO_{mc} = 2.3685$ against the actual $DO_c = 2.6357$ at the 21st day and recovery at $DO_m = 2.4083$ against actual recovery $DO = 2.6677$ on the 26th day of our experimental time before the control. In the same scenario, due to the slight modification of the initial condition, we observed a slight gain in the coordinate of the BOD with $BOD_o(IC = 2.3)$ on the base day and depleted to $BOD_m = 1.2795$ against the actual $BOD = 1.5171$ on the Sixth (6th) day. The BOD_m coordinates due to the modification, decreases drastically from $BOD_m = 1.2795$ on the sixth (6th) day to a converging (saturating) value of $BOD_m=0$ on the 361st day of our experimental time. The full results and discussion is presented in this work.

Keywords: Biochemical-Oxygen Demand, Dissolved Oxygen, Numerical Simulation, Biological Extinction, Model Parameters.

aquatic species within the highly polluted water body. Wastewater emanating from sewage treatment plants often contains an aggregate of organic materials that are decomposed by microorganisms, which use oxygen in the process and the amount of oxygen consumed by these organisms in breaking down the waste is known as the BOD (EPA, 2012). Deoxygenation which refers to the removal of molecular oxygen (O_2) from gases and solvents (Wikipedia, 2022) takes place due to the activities of microorganisms while breaking down organic constituents of the effluent discharged into the river. The contaminated river after undergoing the process of reaeration which is the addition of oxygen through plant respiration/ photosynthesis of aquatic vegetation, dilution from other water bodies and sunlight results in self-purification of contaminated rivers depending on the BOD load in the effluent discharged. In this study, a dynamic mathematical model formulation and numerical simulation approach (Akpodee & Ekaka-a, 2019) will be used to study qualitatively, the interactions between BOD and DO in a highly polluted river due to industrial pollutant discharge. Furthermore, This study will show the impact of pollutant treatment capacity on the BOD and DO level as well as the effect of pollutant treatment capacity increase whether it reduces the pollution in the river leading to a scenario called Self-purification of the river taking into account the deoxygenation and reaeration process. Vital recommendation on recovery of the polluted river in the study area will be suggested. The target of this research work is to draw attention towards the dying condition of river in the Niger Delta and through mathematical model suggest a solution that can be used to control effluent discharge leading to the recovery of polluted waterbodies.

MATHEMATICAL FORMULATION



When water from the river is polluted, the river has an ability to purify itself using some chemical and biological actions. This is known as self-purification. Self-purification can be proved as a good indicator for the status of a river, whenever pollutants are present in a river; two (2) process take place simultaneously namely: Deoxygenation and Reaeration. It is to the extent that these two processes occur that gives a river its self-cleansing property.

We will consider now the case of first-order decay equation of industrial wastes. Equation for two processes is as follows (Kaushik, 2015):

$$\dot{X} = -DC^\tau Y + KT_c^\tau (S - X) \quad (1)$$

$$\dot{Y} = -DC^\tau Y \quad (2)$$

D = decay rate/day

K = proportional coefficient

C = correction coefficient

$$\tau = T - 20$$

X = Dissolved Oxygen (D.O); Y = Biochemical-Oxygen Demand (B.O.D).

T_c = temperature correction coefficient

S = saturation concentration

Analytical Solution

$$\frac{dX}{dt} = -DC^\tau Y + (S - X), \quad X(0) = X_0 > 0 \quad (3)$$

$$\frac{dY}{dt} = -DC^\tau Y, \quad Y(0) = Y_0 > 0 \quad (4)$$

Solving equation (4) by separation of the variable

$$\frac{dY}{dt} = -DC^\tau Y$$

$$\frac{dY}{Y} = -DC^\tau dt$$

Integrating both sides,

$$\int \frac{1}{Y} dY = -DC^\tau \int dt$$

$$\log_e Y(t) = -DC^\tau t + h$$

Taking exponential of both side

$$Y(t) = e^{(-DC^\tau t + h)} = e^{-DC^\tau t} \cdot e^h = A e^{-DC^\tau t} \quad \text{Where } A = e^h \quad (5)$$

$$y(0) = y_0$$

$$y(0) = y_0 = e^{-DC^\tau(0)} \cdot A = A$$

$$y_0 = A$$

Thus

$$y(t) = y_0 e^{-DC^\tau t} \quad (6)$$

Substituting equation (6) into equation (3)

$$\frac{dX}{dt} = -DC^\tau y_0 e^{-DC^\tau t} + KT_c^\tau (S - X)$$



$$\frac{dX}{dt} = -DC^\tau y_0 e^{-DC^\tau t} + KT_c^\tau S - XKT_c^\tau$$

$$\frac{dX}{dt} + XKT_c^\tau = KT_c^\tau S - DC^\tau y_0 e^{-DC^\tau t} \quad (7)$$

The equation is a linear inhomogeneous first order ODE with integrating factor (I.F) of the form

$$\frac{dy}{dx} + p(x)y = Q(x) \equiv \frac{dX}{dt} + p(t)X = Q(t) \quad (8)$$

where $I.F = e^{\int p(x)dx} \equiv e^{\int p(t)dt}$

hence, $I.F = e^{\int KT_c^\tau dt} = e^{KT_c^\tau t}$

multiplying I.F with equation (3.7) we have

$$e^{KT_c^\tau t} \frac{dX}{dt} + e^{KT_c^\tau t} XKT_c^\tau = (KT_c^\tau S - DC^\tau y_0 e^{-DC^\tau t}) e^{KT_c^\tau t} \quad (9)$$

$$\frac{d}{dt} (X e^{KT_c^\tau t}) = KT_c^\tau S e^{KT_c^\tau t} - DC^\tau y_0 e^{-DC^\tau t} \cdot e^{KT_c^\tau t}$$

$$\frac{d}{dt} (X e^{KT_c^\tau t}) = KT_c^\tau S e^{KT_c^\tau t} - DC^\tau y_0 e^{-(DC^\tau - KT_c^\tau)t}$$

Separating variable

$$d(X e^{KT_c^\tau t}) = KT_c^\tau S e^{KT_c^\tau t} dt - DC^\tau y_0 e^{-(DC^\tau - KT_c^\tau)t} dt$$

Integrating both side

$$\int \frac{d}{dt} (X e^{KT_c^\tau t}) = KT_c^\tau S \int e^{KT_c^\tau t} dt - DC^\tau y_0 \int e^{-(DC^\tau - KT_c^\tau)t} dt$$

$$X e^{KT_c^\tau t} = \frac{KT_c^\tau S}{KT_c^\tau} e^{KT_c^\tau t} - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} e^{-(DC^\tau - KT_c^\tau)t} + h_1$$

Dividing through by $e^{KT_c^\tau t}$, we have

$$X(t) = \frac{KT_c^\tau S}{KT_c^\tau} + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau) e^{KT_c^\tau t}} \cdot e^{-(DC^\tau - KT_c^\tau)t} + h_1 e^{-KT_c^\tau t}$$

$$X(t) = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \cdot e^{-(DC^\tau)t} + h e^{-KT_c^\tau t}$$

$$X(0) = X_0$$

$$X(0) = X_0 = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} + h$$

$$h = X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)}$$

$$X(t) = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \cdot e^{-(DC^\tau)t} + \left(X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \right) e^{-KT_c^\tau t} \quad (10)$$

Thus, the predictive model for dissolved oxygen (DO) and Bio-oxygen Demand (BOD) for an arbitrary parameter values are stated as follows

$$X(t) = S + \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \cdot e^{-(DC^\tau)t} + \left(X_0 - S - \frac{DC^\tau y_0}{(DC^\tau - KT_c^\tau)} \right) e^{-KT_c^\tau t}$$

$$Y(t) = y_0 e^{-DC^\tau t}$$



METHOD OF SOLUTION

The numerical simulation aspects using the above solution trajectories of the BOD-DO model will be the core of this studies.

Following Akpodee (2019), when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows:

The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function $f(x, y)$ in the initial value problem

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition $y(x_0) = y_0$

The increment h in x may be changed at each step, but is usually kept constant so that after n steps, we have

$$x_n = x_0 + nh$$

The Runge-Kutta algorithm for the determination of the approximation y_{n+1} to $y(x_{n+1})$ is

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$$

$$k_4 = hf(x_{n+1}, y_n + k_3)$$

The local error involved in the determination of y_{n+1} from y_n is $0(h^5)$

The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z)$$

$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial condition $y(x_0) = y_0$ and $z(x_0) = z_0$

These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.



At the n th integration step, using a step of length h , the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

Where,

$$k_1 = hf(x_n, y_n, z_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2)$$

$$k_4 = hf(x_n + h, y_n + k_3, z_n + K_3)$$

and

$$K_1 = hg(x_n, y_n, z_n)$$

$$K_2 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1)$$

$$K_3 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2)$$

$$K_4 = hg(x_n + h, y_n + k_3, z_n + K_3)$$

As with the Runge-Kutta method, the local error involved in the determination of y_{n+1} from y_n and z_{n+1} from z_n is $O(h^5)$

It is a good numerical analysis practice that in the event of a complex dynamical system that can not admit an analytic solution for sensitivity analysis of interacting variables, we have to adopt an alternative method to study the qualitative behavior of the unique positive co-existence steady-state solution of the DO and BOD as $t \rightarrow \infty$. This is a challenging environmental problem that will be tackle computationally using MATLAB ODE45 numerical scheme.

For the purpose of this study, we have used the following parameter values compressed from the field data stated as follows:

d_1 = BOD decay rate/day

k = Oxygen Supply Proportional coefficient

C = BOD correction coefficient

τ = Oxygen Supply Temperature Difference = $T - 20$

X = DO

Y = BOD

T = Temperature

T_c = temperature correction coefficient for Oxygen Supply

S = saturation concentration for DO

$T_0=20$; $T=28.91$; $\tau = T-T_0$; $T_c=1.0300$; $k=0.01$; $d_1=0.1172$, $X(0)=4.13$, $Y(0)=2.08$; $S=4.50$



Results and Discussion

Table 1.1 Impact of Changing Initial Condition (IC) for DO(IC=4.0) and BOD(IC=2.3) Interactions between a Time Interval of 0(5)90 in Days

Example	Time (Days)	DO (mg/l)	DO _m (mg/l)	BOD (mg/l)	BOD _m (mg/l)
1.	0	4.1300	4.0000	2.0800	2.3000
2.	5.0000	3.2625	3.0465	1.1571	1.2795
3.	10.0000	2.8458	2.5911	0.6445	0.7126
4.	15.0000	2.6743	2.4065	0.3588	0.3968
5.	20.0000	2.6357	2.3685	0.1997	0.2208
6.	25.0000	2.6677	2.4083	0.1112	0.1229
7.	30.0000	2.7356	2.4875	0.0619	0.0684
8.	35.0000	2.8203	2.5850	0.0345	0.0381
9.	40.0000	2.9114	2.6893	0.0192	0.0212
10.	45.0000	3.0032	2.7943	0.0107	0.0118
11.	50.0000	3.0930	2.8967	0.0059	0.0066
12.	55.0000	3.1790	2.9949	0.0033	0.0037
13.	60.0000	3.2608	3.0882	0.0018	0.0020
14.	65.0000	3.3381	3.1762	0.0010	0.0011
15.	70.0000	3.4109	3.2591	0.0006	0.0006
16.	75.0000	3.4792	3.3370	0.0003	0.0004
17.	80.0000	3.5434	3.4101	0.0002	0.0002
18.	85.0000	3.6036	3.4787	0.0001	0.0001
19.	90.0000	3.6600	3.5430	0.0001	0.0001

Table 1.2: Impact of Changing Initial Condition (IC) for DO(IC=4.0) and BOD(IC=2.3) Interactions between a Time Interval of 95(5)180 in Days

Example	Time (Days)	DO (mg/l)	DO _m (mg/l)	BOD (mg/l)	BOD _m (mg/l)
1	95.0000	3.7129	3.6033	0.0000	0.0000
2	100.0000	3.7625	3.6597	0.0000	0.0000
3	105.0000	3.8089	3.7127	0.0000	0.0000
4	110.0000	3.8524	3.7622	0.0000	0.0000
5	115.0000	3.8932	3.8087	0.0000	0.0000
6	120.0000	3.9315	3.8523	0.0000	0.0000
7	125.0000	3.9673	3.8931	0.0000	0.0000
8	130.0000	4.0008	3.9313	0.0000	0.0000
9	135.0000	4.0323	3.9671	0.0000	0.0000



10	140.0000	4.0617	4.0007	0.0000	0.0000
11	145.0000	4.0893	4.0321	0.0000	0.0000
12	150.0000	4.1152	4.0616	0.0000	0.0000
13	155.0000	4.1395	4.0892	0.0000	0.0000
14	160.0000	4.1622	4.1151	0.0000	0.0000
15	165.0000	4.1834	4.1394	0.0000	0.0000
16	170.0000	4.2034	4.1621	0.0000	0.0000
17	175.0000	4.2221	4.1834	0.0000	0.0000
18	180.0000	4.2396	4.2033	0.0000	0.0000

Table 1.3: Impact of Changing Initial Condition (IC) for DO(IC=4.0) and BOD(IC=2.3) Interactions between a Time Interval of 185(5)270 in Days

Example	Time (Days)	DO (mg/l)	DO _m (mg/l)	BOD (mg/l)	BOD _m (mg/l)
1	185.0000	4.2560	4.2220	0.0000	0.0000
2	190.0000	4.2714	4.2395	0.0000	0.0000
3	195.0000	4.2858	4.2559	0.0000	0.0000
4	200.0000	4.2993	4.2713	0.0000	0.0000
5	205.0000	4.3119	4.2857	0.0000	0.0000
6	210.0000	4.3237	4.2992	0.0000	0.0000
7	215.0000	4.3349	4.3118	0.0000	0.0000
8	220.0000	4.3453	4.3237	0.0000	0.0000
9	225.0000	4.3550	4.3348	0.0000	0.0000
10	230.0000	4.3641	4.3452	0.0000	0.0000
11	235.0000	4.3727	4.3550	0.0000	0.0000
12	240.0000	4.3807	4.3641	0.0000	0.0000
13	245.0000	4.3882	4.3727	0.0000	0.0000
14	250.0000	4.3953	4.3807	0.0000	0.0000
15	255.0000	4.4019	4.3882	0.0000	0.0000
16	260.0000	4.4080	4.3952	0.0000	0.0000
17	265.0000	4.4138	4.4018	0.0000	0.0000
18	270.0000	4.4193	4.4080	0.0000	0.0000

Table 1.4: Impact Of Changing Initial Condition (IC) for DO(IC=4.0) and BOD(IC=2.3) Interactions between a Time Interval of 275(5)360 in Days

Example	Time (Days)	DO (mg/l)	DO _m (mg/l)	BOD (mg/l)	BOD _m (mg/l)
1	275.0000	4.4244	4.4138	0.0000	0.0000
2	280.0000	4.4291	4.4192	0.0000	0.0000
3	285.0000	4.4336	4.4243	0.0000	0.0000
4	290.0000	4.4378	4.4291	0.0000	0.0000
5	295.0000	4.4417	4.4336	0.0000	0.0000



6	300.0000	4.4454	4.4378	0.0000	0.0000
7	305.0000	4.4488	4.4417	0.0000	0.0000
8	310.0000	4.4520	4.4453	0.0000	0.0000
9	315.0000	4.4551	4.4488	0.0000	0.0000
10	320.0000	4.4579	4.4520	0.0000	0.0000
11	325.0000	4.4605	4.4550	0.0000	0.0000
12	330.0000	4.4630	4.4579	0.0000	0.0000
13	335.0000	4.4654	4.4605	0.0000	0.0000
14	340.0000	4.4675	4.4630	0.0000	0.0000
15	345.0000	4.4696	4.4653	0.0000	0.0000
16	350.0000	4.4715	4.4675	0.0000	0.0000
17	355.0000	4.4733	4.4696	0.0000	0.0000
18	360.0000	4.4750	4.4715	0.0000	0.0000

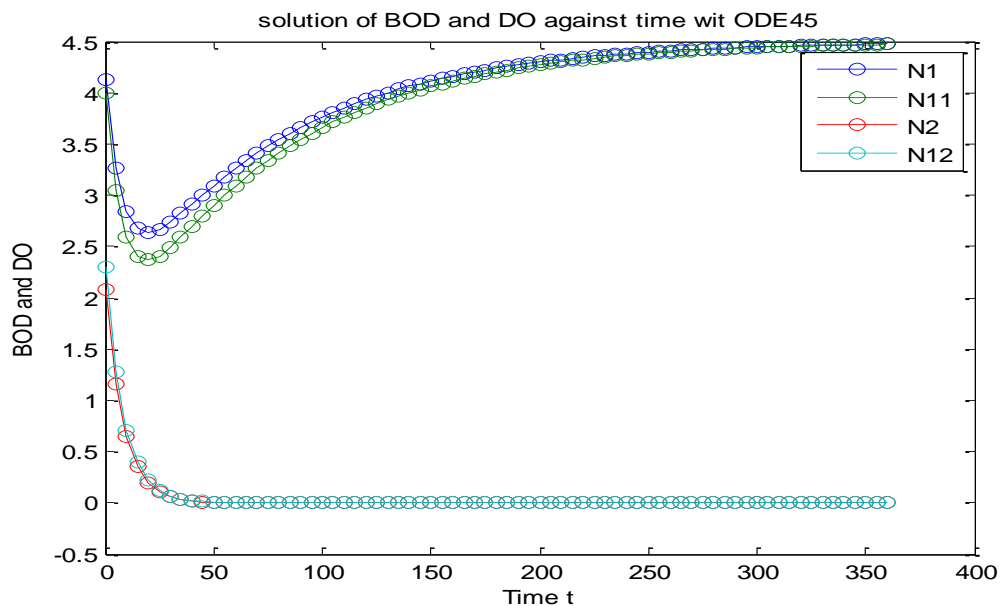


Figure 1.1: Solution Trajectory of The Impact of Changing Initial Condition (IC) for DO(IC=4.0) and BOD(IC=2.3) Interactions between a Time Interval of 0(5)360 in Days

From the result obtained in Table 1.1 – Table 1.4 and Figure 1.1, we observed that when all model parameter values are fixed ranging from the time interval of 0:5:360 in days, the BOD and DO initial values here called the initial condition (IC) on the base day are recorded as $BOD_0 = 2.080$ and $DO_0 = 4.130$ due to pollution of the stream. Furthermore, as a result of the impact of changing the initial condition for DO(IC=4.0) and BOD(IC=2.3) in their interaction, we observed grey area such as the time interval between (0 – 15) days called



the “interval of degradation” with $DO_0 = 4.0$ on the base day up $DO_m = 2.4065$ on the sixteenth (16th) day against the actual the actual $DO = 4.13$ on the base day up to $DO = 2.6743$ on the sixteenth (16th) day before the control. The next grey area is the “interval of active decomposition” which is observed between (10 – 35) days with $DO_m = 2.5911$ on the eleventh (11th) day and $DO_m = 2.5850$ on the 36th day against the actual $DO = 2.8458$ on the 11th day and $DO = 2.8203$ on the 36th day before the control. The DO deficit DO_c is recorded as $DO_{mc} = 2.3685$ at the critical time (days) which shall on the twenty first (21st) day of our experimental time. Moreover, a further grey area captured is the interval of active recovery which ranges from (35 – 150) days has $DO_m = 2.5850$ on the 36th day and $DO_m = 4.0616$ on the 151st day been the “point of inflexion” against the actual $DO = 2.8203$ on the 36th day and $DO = 4.1152$ on the 151st day been the actual value of the point of inflexion before the control. Here, an important region or grey area called the interval of complete saturation has $DO_m = 4.0616$ at the point of inflexion on the 151st day and a saturated value of $DO_m = 4.4715$ on the 361st day of our experimental time against the actual $DO = 4.1152$ on the 151st day and $DO = 4.4750$ on the 361st day before control. This observation due to a slight change in the initial condition of DO ($IC = 4.0$) and BOD ($IC = 2.3$) portrays a threat to the aquatic environment as the grey areas indicate a depletion in the coordinates of the DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a critical $DO_{mc} = 2.3685$ against the actual $DO_c = 2.6357$ at the 21st day and recovery at $DO_m = 2.4083$ against actual recovery $DO = 2.6677$ on the 26th day of our experimental time before the control. In the same scenario, due to the slight modification of the initial condition, we observed a slight gain in the coordinate of the BOD with BOD_0 ($IC = 2.3$) on the base day and depleted to $BOD_m = 1.2795$ against the actual $BOD = 1.5171$ on the Sixth (6th) day. The BOD_m coordinates due to the modification, decreases drastically from $BOD_m = 1.2795$ on the sixth (6th) day to a converging (saturating) value of $BOD_m = 0$ on the 361st day of our experimental time. This observation is consistent and vital for environmental decision and policy making which will improve the scope of protecting the water bodies and aquatic environment as a result of pollution. The information discussed here captures grey area of vulnerability of the aquatic environment and will serve as a valuable tool that may be utilized by environmental protection agency and other related bodies concerned with pollution for monitoring, intervention and mitigation.

CONCLUSION

Using the analytical method to study the qualitative characterization of the dynamic system is tasking and did not give an early insight of the expected results and also involved some errors due to approximation, we introduced the core method for this work which is the computational method. Through this method, we were able to predict the relative abundance of the BOD - DO and the possible effects on aquatic environments in their



interactions over a period of 360 *days at an interval* of 0: 5: 360 in days using MatLab ODE45 numerical scheme. The grey areas were observed as well as the relative abundance of both coordinates and a point of recovery in the DO coordinates with its DO deficit up to its complete saturation.

The differential equation model for effluents discharged used in this work is very effective in the determination and prediction of the relative abundance of the BOD-DO coordinates over a period of time.

RECOMMENDATION

To researchers who may want to do similar works as this, we recommend that:

The impact of the growth rate should be checked using computational method.

The impact temperature variation should be carried out using computational method.

Similar method should be adopted to study the qualitative behavior of other physiochemical properties over time.

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