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EFFECT OF MISCONCEPTIONS IN LEARNING SCHOOL GEOMETRY ON STUDENTS' ACHIEVEMENT IN GEOMETRY: FOCUS ON MISCONCEPTIONS ABOUT LINES AND ANGLES.

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ABSTRACT

Geometrical knowledge is essential for mathematical mastery of secondary school students. In fact, evidence from literature revealed that geometrical ability of student is determined by their level of conceptions and misconceptions about geometry. In order to ascertain the effect of misconceptions in learning geometry among secondary school students, this study examined the effect of misconceptions about lines and angles in geometry class on students'

INTRODUCTION

Understanding concepts in a subject is very important, if not necessary, for the learning of this subject. The kind of school geometry being learnt in Nigeria and many other countries of the world has been based on the formal axiomatic geometry which Eulid of Alexandaria created over 2000 years ago. This kind of geometry is based on axioms, postulates, definitions, theorems and proofs. Various forms of misconceptions or conceptual misunderstandings in the learning of this great subject have been reported in the literature. A misconception is the inability of students or learners to really understand the ideas or philosophical notions held by the concept being learnt. The results of these misconceptions have been poor learning of geometry, leading to under-achievement. The current researcher got his inspiration to carry out an investigation into the effect of misconceptions in learning school geometry on students' achievement in geometry from this sad situation.

Research literature is replete with reports on various forms of misconceptions experienced by learners in their learning of school geometry. The most commonly reported learning difficulties in the literature are: misconceptions, imprecise terminology , identification, classification of basic

achievement in geometry. It was a quasi-experimental research design guided with five (5) research questions and research hypotheses. Simple random sampling technique was used to select two hundred and ten (210) participants from three (3) senatorial zones in the study area. The participants were divided into two (2) groups- control (104) and experimental (106). The study used Geometry Concept Test (GCT- measure of how much conception and misconception) and Geometry Performance Test (GPT measure of the level at which geometry is learnt generally) as instruments for data collection. The instruments yielded reliability coefficient of 0.85 and 0.81 for GCT and GPT respectively after using Pearson's Product Moments Correlation Coefficient technique (PPMC). Frequency and percentage was used for answering research questions while t-test analysis was used to test hypotheses at 0.05 level of significance. The study revealed that greater number of students in the control than treatment group developed incorrect concepts of geometry; small number of control group developed correct concepts of geometry; majority of treatment group developed correct concepts of geometry; very few number of control group had correct concepts of geometry; development of correct or incorrect concepts by participants was attributable to the kind of teaching practice in use; and that achievement of treatment group was better than control group. The study further indicated significant effect of misconceptions of diagonals linked to students' achievement in geometry; misconceptions about parallel and perpendicular lines had significant effect on students' achievement in geometry; and students' achievement in geometry was significantly affected by misconceptions about the general angle; as well as significant effect of students' misconceptions about the sum of angles in a triangle on students' achievement in geometry. Based on the findings, it was recommended among others that geometry teachers should de-emphasize classroom instructions which make students passive and emphasize instructions which adopt practical approaches to teaching and use relevant instructional materials; and that geometry teachers are expected to patronize workshops, conferences and seminars, and other relevant means for further self-development on the job.

Keywords: Geometry, misconception, secondary students, parallel, triangle.

shapes, properties of shapes, classifications of shapes, parallel and perpendicular lines, the concept of angles, angle sum of a triangle, and proof writing (Mayberry, 2003; Burger and Shaughnessy, 2006; Howie, 2001; Roux, 2003; French, 2004; Feza and Webb, 2005; Siyepu, 2005). The common misunderstandings in learning demonstrated by senior secondary school students in Taraba State and elsewhere form important subjects of investigation in this study, especially misunderstandings having to do with the concept of lines (diagonals, parallel and perpendicular lines).

A very important aspect of geometry learning is accomplishing tasks. Given a geometrical task to accomplish, students try to understand it through reasoning, and by reasoning too, propose possible method(s) of solving the problem and finally resolve it. This is in line with scientific method. With a task before him/her, a scientist raises questions in his/her bid to gain a proper understanding of it, find answers to these questions which may or may not be answers to the task. In certain cases he/she may have to experiment. From the fore-going, it can be seen that tackling a geometrical (mathematical) task is in line with scientific methods. This is because; the behavior of an accomplisher of a geometrical task is, in many respects similar to that of scientist. Wilson (2005) observes that students' geometrical task may be modest; but if it challenges their curiosity and brings into play their inventive faculties, and if they solve it by their own means, they may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprints on mind and character for a lifetime. The tension so experienced is likely due to misunderstanding of concepts which is the source of inspiration for the researcher to embark on the current investigation.

Susceptibility, according to De Jager-Haum (2000), deals with the ease with which an individual is upset. Therefore geometry students being at a susceptible age mean they are at an age they are easily upset. So grooming them up at such an age such that they are able to solve geometrical problems on their own and by their own means, will develop in them a propensity or learning (inclination) for mental work.

Solving problems (task-accomplishment) has a special place in the study of geometry. A primary goal of teaching and learning geometry is to develop the ability of students to carry out a wide variety of complex geometrical tasks (Minstrel, 2009). Mathematicians have traced the role of doing tasks in school geometry and have illustrated a rich history of it. To many geometrically literate people, geometry is synonymous with carrying out tasks-doing word problems, creating patterns, interpreting figures, developing geometric constructions, proving theorems, etc- (all involving reasoning and requiring conceptual understanding). In fact, learning to solve problems is the principal reason for studying mathematics and geometry, (U. S. National Council of Supervisors of Mathematics, 2008). This is a motivating factor for the current researcher to embark on the investigation of conceptual understandings and misunderstandings involved in geometry learning, especially concerning the concept of line.

In Nigeria, and indeed in all countries, mathematics in general and geometry in particular, is regarded as a cardinal factor in the nation's scientific and technological advancements because of its useful links to many other fields of human endeavour (Federal Ministry of Education, 2005). Students' competencies in geometry have been closely linked to their geometric thinking levels and to their understandings of concepts in geometry (French, 2004; Van Hiele, 1986). The linkage between geometry learning and conceptual understanding, being threatened by misunderstanding of concepts forms the basis for the focus of this study on the effect of conceptual misunderstandings on students' geometry achievement.

Trends in achievement in mathematics (with geometry forming a very significant part), over the past few years, have indicated that male and female learners achieve differently. A look at the summary of students' achievements taken from the 17th, 18th, 19th, and 20th WAEC state committee meetings, agenda and papers for the years 2010, 2011, 2012 and 2013 respectively for Taraba State discloses that:

1. Percentages of passes at credit levels (i.e. grades 1-6) for general mathematics and for females are lower than those for males. For instance, we have 22.5% for females against 24.5% for males.
2. Further, disparities between the percentages for males and females are even much more pronounced with further mathematics than they are with general mathematics. For instance, we have 9.0% for females as against 22.2% for males in 2011; and 10.1% for females as against 33.05% for males in 2010. Why the differential achievements? Are there more male students with conceptual misunderstandings than females or more females than males with this problem?

   This forms a basis for the researcher to investigate whether gender has an effect on conceptual understanding.

Geometry is a mathematical science which deals with the properties and relations of lines/surfaces/solids, etc in space (De Jager-Haum, 2000). More simply put, geometry is an aspect of mathematics which deals with the properties of points, lines, surfaces and solids, either under classical Euclidean assumptions, or (in the case of Elliptic, Hyperbolic, etc geometry), involving postulates not all of which are identical with Euclid's. The current

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investigation explores the understandings of concepts in geometry and whether their misunderstanding has effect on achievement in the subject.

To achieve means to succeed (in doing something) or to reach (a goal), achievement therefore is a measure of success in doing something. In Education, achievement is measured by the scores testees are able to make in a test. Having this in mind, the geometry achievement of students is measured by their scores in a geometry test. In this investigation, the effect of misconceptions in geometry (especially line geometry) on students' geometry achievement was explored.

Statement of the problem

The learning of school geometry in Nigeria has, for a long time, been based upon the formal axiomatic geometry which Euclid created over 2000 years ago (Balasa, 2021). In his era, Eulid's logical construction of school geometry with its axioms, postulates, definitions, theorems and proofs was, indeed, an admirable achievement (Van Hiele, 1999). However, Van Hiele (1999) expressed the feeling that school geometry, presented and learnt in the traditional Euclidean fashion assumes that school children also think at the formal deduction level. The Hiele's argument here can be understood better in the light of the following statement: that for students to learn school geometry in this classical Euclidean fashion, they should strive to not only understand but also translate axioms, postulates definitions and theorems into constructible geometry. But not all the students are at this formal deduction stage and not all have developed the requisite conceptual understanding to carry out this exercise. The thought processes injected into the learning of this kind of geometry and the development of conceptual understanding required for such learning may not be the same for all the students at the same time. Worried about students' weaknesses in their learning of school geometry as reported by WAEC Chief Examiner (2015) and by their poor performance in the subject (WAEC, 2010; 2011), the current researcher embarked on the study of the relationship between misconceptions in learning school geometry and students' geometry achievement. Put as a question, is there any effect of misconceptions in learning school geometry on students' achievement in the subject?

Purpose (Objective) of the Study

The purpose of this study generally was to determine the effect of students' misconceptions about learning concepts in school geometry on their achievement in the subject, with special attention given to misconceptions about lines and angles. In specific terms, the purpose of the study was to determine the:

1. Effect of misconceptions about diagonals on students' achievement in geometry;
2. Effect of misconceptions about parallel lines on students' achievement in geometry;
3. Effect of misconceptions about perpendicular lines on students' achievement in geometry;
4. Effect of misconceptions about the concept of the general angle on students' achievement in geometry; and
5. Effect of misconceptions about the angle sum of a triangle on students' achievement in geometry.

Significance of the Study

This investigation is significant to four (4) different sets of people: geometry students, geometry teachers, geometry educators, (i.e. teachers of teachers of geometry), and to curriculum planners.

The study is significant to geometry students because it enhances the development of imaginative thinking and conceptual understandings in them. Many of the students' activities in the study and indeed geometrical activities of other students generally are so prepared that students are required to visualize geometrical figures and to use properties of these visualized figures to solve problems. By so doing, an opportunity is created for these students to develop imaginative and conceptual understandings, through critical and creative thinking, in order to understand geometric phenomena. Hence imaginative reasoning is enhanced and conceptual understandings are promoted. Enhancement of both imaginative and conceptual understandings leads to development of self confidence. This is because; students respond confidently once imaginative and conceptual understandings are developed in them. Confident responses are signs of real understandings of concepts in geometry.

This study is also significant to geometry teachers because from it, these teachers will be exposed to various forms of misconceptions developed by students as they learn school geometry. Misconceptions by students impede their learning of geometry and lower their achievement in the subject. Such information will guide these teachers to select appropriate teaching methods and instructional materials, and organize instructions such that misconceptions by students are lessened.

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To geometry educators, gathered data from this investigation will guide them in selecting, organizing and presenting materials to geometry teachers on the best way to select geometry content in which misconceptions are prevalent among students and suggest suitable methods of presenting the content to facilitate conceptual understanding. This information can also be used to educate would-be geometry teachers in preparation in schools.

This investigation will also serve as a guide to curriculum planners for not only determining but also for sequencing materials to be learnt by students whose concepts are inter-connected. This will go a long way in promoting conception and lessening misconceptions. The study is equally likely to serve as a ground from which further researches on misconceptions in learning school geometry will spring.

Scope of the Study

This study was restricted to exploring the Effect of misconceptions that students develop about concepts in geometry while they learn school geometry on their achievement in the subject. Specifically, the study was limited to the following:

i) Senior Secondary two classes of students in the state of Taraba;

ii) Geometrical tasks relevant to SS2, which include:

   a) Recognition of geometrical lines (diagonals, parallel, perpendicular);
   b) Describing properties of lines;
   c) Establishing relationships between properties and lines;
   d) Understanding the role of postulates, axioms, theorems and proofs, and
   e) Formal reasoning about geometrical systems.

Research Questions

The following research questions guided the conduct of the study:

1. What is the effect of misconceptions about diagonals on students’ achievement in geometry?
2. What is the effect of students’ misconceptions about parallel lines on their achievement in geometry?
3. What is the effect of students’ misconceptions about perpendicular lines on their achievement in geometry?
4. Is there any effect of students’ misconceptions about the concept of the general angle on their achievement in geometry?

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5. What is the effect of students' misconceptions about the angle sum of a triangle on their achievement in geometry?

Research Hypotheses

The following statements of hypotheses were formulated and tested at the 5% (0.05) level of significance:

1. There is no significant effect of students' misconceptions about diagonals on their achievement in geometry.
2. There is no significant effect of students' misconceptions about parallel lines on their achievement in geometry
3. There is no significant effect of students' misconceptions about perpendicular lines on their achievement in geometry.
4. Students' misconceptions about the concept of the general angle do not significantly affect their achievement in geometry
5. There is no significant effect of students' misconceptions about the angle sum of a triangle on their achievement in geometry.

Methods

The investigator used a quasi-experimental design to explore the effect of misconceptions in learning school geometry on students' achievement in the subject. The researcher took and used a random sample of two hundred and ten (210) senior secondary two (SS2) students for the study. Also, three (3) out of one hundred and eighteen (118) public senior secondary schools were taken at random and used. To carry out both selections (i.e. selection of schools and students), the researcher used cluster random sampling technique, that is to say that all the 118 public senior secondary schools in the state of Taraba were clustered into three on the basis of the following geo-political sub-divisions: Taraba North, Taraba Central and Taraba South senatorial zones.

From each of these 3 clusters, a school was randomly chosen, using the lottery technique, and each of the chosen schools was named schools 1, 2 and 3. For the selection of research participants, the simple random sampling technique was used to take two intact classes of SS2 students from each of the 3 randomly selected schools. The entire students in the randomly selected intact classes participated in the study. The distribution of students in the 3 randomly chosen schools is displayed in the table below:

Table 1: Distribution of participating students According to Sampled schools

Schools		1		2		3	Total
Classes	A	B	C	D	E	F	
Males	20	19	21	20	20	17	117
Females	16	15	14	15	17	16	93
Total	36	34	35	35	37	33	210

In the next stage of researching, the investigator randomly assigned the participating students to either of the following groups: treatment (experimental) or control (usual lecture). Accordingly, and through sampling by lottery, the classes labeled A, D and F (in Table 1 above), were assigned to the treatment group while B, C and E had their assignment done to the control group. The treatment or experimental group received instructions from a geometry teacher with wealth of training and experience, using instructional materials and exemplary classroom practices. Students in the control group simply received instructions through the normal lecture. After treatment and lecture, two tests called the Geometry Concept Test (GCT) and the Geometry Performance Test (GPT) were developed and administered. The test administration enabled data to be gathered in two respects: Data from the GCT helped in determining how much conception and misconception was developed by the participating SS2 students as they learnt school geometry. Data from the GPT on the other hand, helped in determining the level at which geometry was learnt generally by the students, depending on the levels of conception and misconceptions developed while learning. The two instruments, GCT and GPT had their validity determined by 3 mathematics teachers, one each from the three participating schools and by 3 seasoned mathematics (geometry) educators, one each from the College of Education, Zing; Taraba State University (TASU), Jalingo; and Modibbo Adama University (MAU), Yola. An estimation of the reliability of the two tests was done by using the test-retest reliability approach. Pearson's Product Moments Correlation Coefficients Method (PPMC) was used to obtain reliability coefficients of 0.85 and 0.81 for GCT and GPT respectively. These coefficients are indications that the instruments are really reliable.

Experimental Procedure

In order for the researcher to obtain data on which to base investigational findings, research participants were given two different kinds of instruction: participating students in the Experimental Group (EG) received instructions in which exemplary classroom practices were used with rich instructional materials to facilitate conception of geometrical ideas. On the other hand, their counterparts in the Control Group (CG) had their instructions via the conventional lecture method. Experimental teachers (teachers in the experimental classrooms) were groomed, enabled and advised to use the said exemplary classroom practices and a variety of instructional materials to maximize opportunities for development of conception and to lessen misconceptions. In particular, participating teachers were trained to be able to use instructional materials that will enable students make a variety of geometrical constructions and models from which to learn the concepts and properties of symmetry of lines, parallelism, perpendicularity and diagonals. In order for these quasi-experimental teachers to be able to properly handle this treatment, a two week training exercise was organized for them. The research conditions, including the proper and skilful handling of instructional materials were properly explained. The framework for thinking about exemplary classroom practices was exhaustively explained. Participating teachers in the control classrooms were simply groomed to be able to handle lecture as instructional method. Also, participating SS2 students in both control and treatment classrooms were subjected to instructions for four (4) weeks. The four week session of instructions was followed by a session of test administration in which two tests were administered. The administered tests were:

1. The Geometry Concept Test (GCT)- an instrument which was used to measure the level of geometry concepts developed by students and to detect misconceptions; and
2. The Geometry Performance Test (GPT) – an instrument which was used to measure the level of geometry content learnt by the students (i.e. achievement level in geometry). The marking guide (scheme) prepared by the researcher was used to mark students' answer scripts. Test scores generated data that were analyzed by using mean and standard deviation.

The research questions which guided the conduct of the study were answered by these means and standard deviations. The t-test statistic was used for testing the stated null hypotheses for significance at the 0.05 (5%) level of significance.

Results

In this sub-section of the research report, results are presented and their analysis made. Tables are used for presenting the results. Each table is presented by a research question and information in the table answers the research question. Following the research question is a hypothesis for testing at the 0.05 (5%) significance level. This hypothesis testing was done by using the t-test statistic. Following the summary of each result is a statement which accepts or rejects the stated null hypothesis.

Research Question 1 (a)

What are students' misconceptions about the diagonals of geometrical figures?

Students were asked to state the number of sides and diagonals possessed by the following figures:

Fig 1 (a)

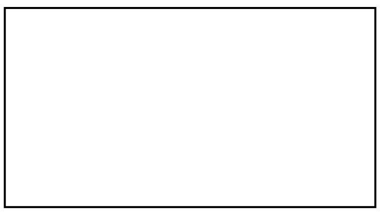
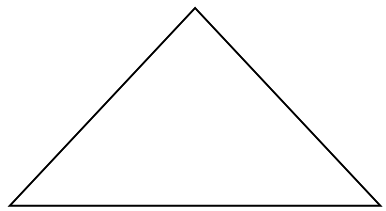


Fig. 1 (b)



Their responses fell within the following categories:

Fig. 1(a)

i) 4 sides, no diagonals
ii) 4 sides, 4 diagonals
iii) 4 sides, 2 diagonals

Fig. 1(b)

i) 3 sides, 3 diagonals
ii) 3 sides, 1 diagonal
iii) 3 sides, no diagonals

Results showing number of students with responses in the 3 categories for each of fig. 1(a) and fig. 1(b) are shown in tables 2(a) and (b) below:

Table 2(a): Number of Students with Misconceptions about Diagonals
Fig. 1(a)

No. of students Groups	4sides, no diagonals	4sides, 4 diags.	4sides, 2diags.	Total
Treatment	18	14	72	104
Control	47	34	25	106
Total	65	48	97	210

Table 2(b): Number of Students with Misconceptions about Diagonals (Fig. 1(b)

No. of students	3sides, 3 diagonals	3sides, 1 diag.	3sides, no diag.	Total
Groups				
Treatment	10	26	68	104
Control	50	37	19	106
Total	60	63	87	210

From table 2a (above), majority of the 210 students who participated in the study, 113 of them, (representing 53.81%) had one form of misconception about diagonals or the other. Only 97 of these students, representing 46.19%, developed the correct concept of the diagonal. Further, up to 72 students in the treatment class, representing 69.23% had correct concept of diagonals. In contrast, merely 25students in the control group, representing 23.58% had this correct concept. Conversely, 81 of the 106 control students, representing 76.42% had misconceptions about diagonals in contrast with merely 32 treatment students, representing just 30.77% with such misconceptions.

Trends in table 2(b) are similar to those in table 2(a). Greater number of the 210 participating students, 123 of them, (representing 58.57%) developed misconceptions about diagonals. Merely 87 of these students, representing 41.43%, developed the correct concept of diagonals. 68 of these 87 students with correct concept of diagonals, representing 65.38%, were in the treatment classroom. A mere number of 19 control students, representing just 17.92%, managed to develop the correct concept of diagonals. Misconceptions about diagonals were developed by an all high number of 87 control students, representing 82.08%. A number as small as 36 of the 104 treatment students, representing just 34.62%, had these misconceptions.

Research Questions 1(a) and (b)

What are students' misconceptions about parallel and perpendicular lines?

Students were asked to find measures of angles labeled a, b, c, and d as indicated in fig. 2 below; Where PQ and RS are parallel lines and JK is a transversal:

Fig. 2

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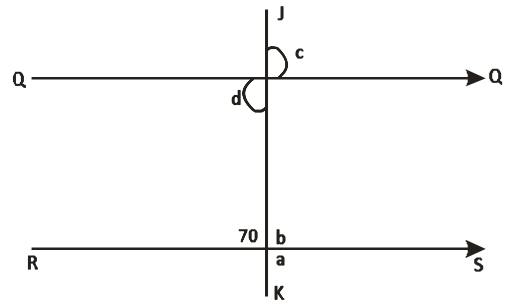

To respond to the question above, students would require using the angle relationships of parallel lines and the transversal which cuts across them. Responses of students were reflections of difficulty in developing and gaining acquaintance with the terminology relating to parallel lines and transversals. In other words, students' responses showed misconceptions about such terminologies relating to parallel lines and transversals as corresponding angles, vertically opposite angles, alternate angles, co-interior angles, angles on a straight line (adjacent or supplementary angles). From their responses, table 3 below was drawn to depict number of students with misconceptions about terminologies relating to angle relationships of parallel lines and transversals:

Table 3: Number of students with misconceptions about parallel and perpendicular lines:

No. of students Groups	Alt. <'s	Co-int. <'s	Vert. Opp. <'s	Corr. <'s	Sup. <'s	NM	Total
Treatment	6	3	4	4	5	82	104
Control	28	16	12	14	13	23	106
Total	34	19	16	18	18	105	210

Results from the table above (table 3), reveal that participating students developed various misconceptions about terminologies relating to angle relationships of parallel lines and transversals in the following respects. (i) Misconceptions about alternate angles (alt. <'s) developed by 34 students (6 in the treatment class and 28 in control); (ii) Misconceptions about co-interior angles (Co-int. <'s) developed by 19 students (3 in treatment class and 16 in control); (iii) Misconceptions about Vertically opposite angles (Vert. Opp. <'s) developed by 16 students (4 treatment, 12 control); (iv) Misconceptions about Corresponding angles (Corr. <'s) developed by 18 students (4 treatment, 14 control); and (v) Misconceptions about Supplementary angles (Sup. <'s), same as angles on a straight line, developed by 18 students (5 treatment, 13 control). Also, from the table,

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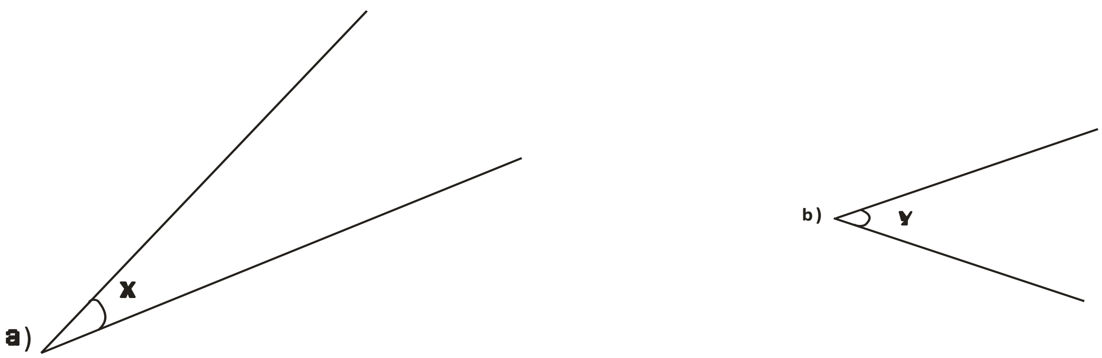
82 treatment students, representing 39.05% of all the 210 participating students developed correct concepts about terminologies (had no misconceptions) relating to angle relationships of parallel and perpendicular lines. In sharp contrast, merely 23 control students, representing just 10.95%, managed to develop these correct concepts:

Research Question 1(d)

What are students misconceptions about the concept of the general angle?

Participating students in this study were asked to indicate in writing, which of the angles in the figure below is smaller than or larger than the other, and to give reason(s) for their answer(s):

Fig. 3



Their responses were grouped into four (4) as follows:

i) The measure of the angle x is larger than that of y because the arms or rays are longer (longer rays, larger angles).

ii) The measure of the angle y is smaller than that of x because angle y has shorter rays (shorter rays, smaller angles).

iii) The sizes are not known, so one cannot say which is smaller or larger than the other.

iv) The angle labeled x is smaller than the one labeled y.

Results indicating number of students with misconceptions about the general angle in the various categories of responses are presented in table 4 below:

Table 4: Number of students with Misconceptions about the general angle:

No. of students	Longer rays,	Shorter rays,	Sizes Unknown	Correct	Total

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Groups	Larger angles	Smaller angles		Concept	
Treatment	13	8	4	80	104
Control	37	28	21	20	106
Total	49	36	25	100	210

From table 4 (above), the following results are obviously seen:

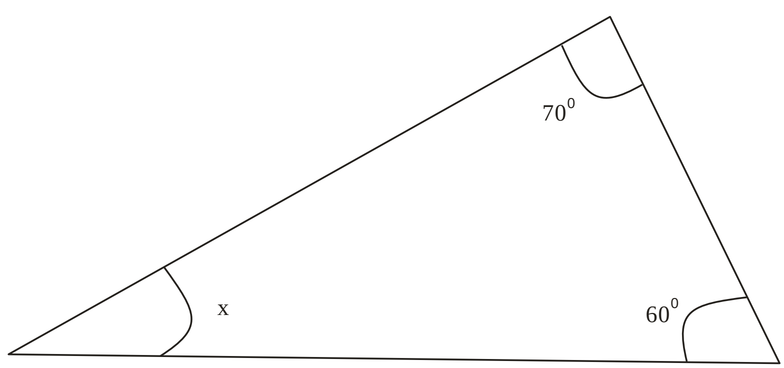
Of the 210 SS2 students who participated in the study, 110, representing 52.38% had misconceptions about the concept of the general angle. On the other side of the balance, 100 of these students, representing 47.62% developed correct concept about the angle. It may be interesting to the reader to note that 80 of the 100 students with correct concept of the general angle, representing 80%, participated as treatment subjects. A very small number of the control subjects, 20 to be precise, representing 20%, managed to develop this correct concept. This means that majority of the students who worked in the control classroom failed to develop the correct concept of the general angle. Numerically, 86 of these control students, representing 81.13% of the students in that group, had this failure. Contrastingly, merely 24 students, representing just 23.08% of students in the treatment group, had misconceptions about the concept of the angle.

Research Question 1(f)

What are students' misconceptions about the angle sum of a triangle?

Students were engaged with the tasks of finding the size of the third sides of a number of triangles in which the sizes of the other two sides were known, such as one in fig. 4 below: Find the size of the angle marked x

Fig. 4



From their responses, it was evident that students generally lacked knowledge of the angle sum of a triangle. Results are presented in table 5 below:

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Table 5: Students' Misconceptions about angle sum of a triangle:

No. of students Groups	Correct Concepts	Incorrect Concepts	Total
Treatment	71	33	104
Control	21	85	106
Total	92	118	210

To respond to questions correctly such as one involving fig. 4 above, the students needed to have an adequate understanding of the theorem which states that the angle sum of a triangle is 180°. That is to say that the sizes of three angles of a triangle add up to 180°. So to get the size of the third angle, the sizes of the two known angles are added together and their sum is subtracted from 180°. However, as can be seen from table 5, responses of the majority of the participating students were reflections of lack of knowledge of the angle sum of a triangle. Clearly, it is seen from the table that only 92 students in all, representing 43.81% of all participating students, had correct concept of the angle sum of a triangle. Quite a larger number of them, precisely 118, representing 56.19%, had incorrect concept of this sum. Majority of students with correct concept, 71 of them in number, representing 68.27% were in the treatment group. Only 33 treatment students, representing just 31.73%, had incorrect concept of the sum of angles of a triangle. With respect to the control students, only a few of them, 21 to be precise, representing merely 19.81%, had this correct concept. A vast majority of the control subjects, numbering 85 and representing 80.19%, were not able to develop the correct concept of the angle sum of a triangle.

Research Question 2:

What is the effect of students' misconceptions about diagonals on their achievement in the Geometry Performance Test (GPT)?

Table 6 below depicts the effect of students' misconceptions about diagonals on their achievement in the GPT through their mean and standard deviation scores:

Table 6: Mean and Standard Deviation Scores of participating SS2 students with misconceptions about diagonals

Groups	N	Mean(x)	Standard Deviation (t)
Treatment	104	8.4732	68.072
Control	106	7.728	4.5264
Difference		0.7452	1.5456

Bearing in mind the content of table 6 (above) it is clearly seen that students in the treatment group have their mean score as 8.47 and their standard deviation is 6.07. On the other hand, the mean score of the control students stands at 7.73 and their standard deviation is 4.53. It is easy to see from these figures that there is a difference of 0.75 between the means of the two groups and a difference of 1.55 between the standard deviations of these groups, indicating the supremacy or superiority of the achievement of the treatment students over that of the control group on the GPT. Note that it was noted in tables 2a and 2b that there were greater numbers of students in the control group with misconceptions about diagonals than there were in the treatment group. This informed the lower achievement made by the control students made on the GPT reported in table 6.

H_{01}: there is no significant effect of students' misconceptions about diagonals on their achievement in geometry

Table 7 below gives the summary of t-test analysis of SS2 students' achievement.

Table 7: Summary of t-test Analysis of SS2 students' Achievement with Misconceptions about Diagonals

Groups	N	Mean	Std Deviation	t-cal	t-crit	df	Inference
Treatment	104	7.27	3.87	19.6	1.55	208	Significant
Control	106	6.50	2.22				

It is evident from table 7 that students in the treatment group where there were exemplary classroom practices and less misconceptions about diagonals, had higher mean score than that of students in the control group where there was lecture, passivity or more misconceptions about diagonals. More so, the critical value of t is far less than the calculated value of t. In other words, t-crit (1.55) is less than t-cal (19.6). This means that computed t-value of 19.6 far exceeds that of the tabulated t-value of 1.55. On this ground, the stated null hypothesis is rejected. This disconfirms the statement that there is no significant effect of misconceptions about diagonals on students' achievement in geometry. Misconceptions about diagonals do have significant effect on students' achievement in geometry.

Research Question 3:

What is the effect of students' misconceptions about parallel and perpendicular lines on their achievement in the GPT?

Table 8 below presents the effect of students' misconceptions about parallel and perpendicular lines on their achievement in the GPT

Table 8: Mean and Standard Deviation Scores of Participating SS2 students with Misconceptions about Parallel and Perpendicular Lines

Groups	N	Mean(x)	Standard Deviation
Treatment	104	11.77	5.50
Control	106	10.05	4.94
Difference	02	1.72	0.56

From table 8 (above), entries reveal that there exists a mean difference of 1.72 and a difference in standard deviation of 0.56 between the treatment and control students. Students in the treatment class have higher mean and standard deviation than those in the control class. It was noted earlier too, (in table 3) that more control students had misconceptions about parallel and perpendicular lines than treatment. As such their achievement on the GPT got lowered by the effect of these misconceptions.

H_{02}: There is no significant effect of students' misconceptions about parallel and perpendicular lines on their achievement in geometry.

Table 9 below contains the summary of the t-test analysis of SS2 students' achievement reflecting their misconceptions about parallel and perpendicular lines.

Table 9: Summary of t-test analysis of SS2 students Achievement reflecting their Misconceptions about Parallel and Perpendicular Lines:

Groups	N	Mean (x)	Std Deviation	t-cal	t-crit	df	Inference

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Treatment	104	5.17	3.3	22.9	1.55	208	Significant
Control	106	3.90	2.74				

With evidence in table 9, the mean score and standard deviation of students in the treatment class are higher than they are in the control class (3.9 and 2. 74) respectively. Further, the calculated value of t (22.9) far exceeds that of the critical t (1.55). This forms a basis for the rejection of the stated null hypothesis. This rejection confirms the assertion that misconceptions about parallel and perpendicular lines have significant effect on students' achievement in geometry

Research Question 4:

What is the effect of misconceptions about the general angle on students' achievement in the Geometry Performance Test?

Table 10 below gives information about the effect of students' misconceptions about the general angle on their achievement in the GPT as represented by the mean and standard deviation of their scores:

Table 10: Mean and Standard Deviation Scores of Participating SS2 students reflecting their Misconceptions about the General Angle

Groups	N	Mean(x)	Standard Deviation
Treatment	104	12.01	6.02
Control	106	11.12	5.01
Difference	002	0.89	1.01

Evidently, from table 10, the mean score of students is higher in the treatment class (12.01) than it is in the control class (11.12). Also, standard deviation is higher in the treatment group (6.02) than it is in the control (1.01). This is attributable to misconceptions developed by majority of the control students as noted in table 4. Hence their lower mean score and lower standard deviation.

H_{o3}: Students' Misconceptions about the general angle have no significant effect on their achievement in geometry.

Table 11 below is a summary of the t-test analysis of SS2 students' achievement reflecting their misconceptions about the general angle.

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Table 11: Summary of t-test analysis of SS2 students' achievement reflecting their misconceptions about the general angle:

Groups	N	Mean (x)	Std Deviation	t-cal	t-crit	df	Inference
Treatment	104	2.67	0.80	20.40	1.55	208	Significant
Control	106	1.40	0.24				

From table 11 above, it becomes evidently clear that treatment students are superior to control students in terms of mean and standard deviation scores. Again, the calculated value of t (20.40) is many times greater than the critical t-value (1.55). Under circumstances such as this, the null hypothesis is to be rejected. Our hypothetical assertion that "misconceptions about the general angle have no significant effect on students' achievement in geometry" was therefore rejected. With this rejection, it becomes established that students' misconceptions about the general angle really have significant effect on students' achievement in geometry.

Research Question 5:

What is the effect of students' misconceptions about the angle sum of a triangle on their achievement in geometry?

Table 12 below gives information about the effect of students' misconceptions about the angle sum of a triangle on their achievement in the GPT

Table 12: Mean and Standard Deviation Scores of SS2 students reflecting their misconceptions about angle sum of a triangle

Groups	N	Mean(x)	Standard Deviation
Treatment	104	6.05	5.22
Control	106	4.78	2.04
Difference	002	1.27	3.18

Like with other tables, the content of table 12 shows differences in mean and standard deviation of 1.27 and 3.18 between treatment and control students respectively. Also, it has been noted in table 4 on page 15 that majority of the control students (85 of the 110 of them), had misconceptions about the concept of the general angle. A factor to be used to explain their low achievement on the GPT is therefore effect of these misconceptions.

The treatment students on the other hand had superior achievement because majority of them, 80 out of 100, had correct concept.

H_{04}: Students' misconceptions about the angle sum of a triangle do not significantly affect their achievement in geometry.

Table 13 below contains information which gives the summary of the t-test analysis of SS2 students' achievement in geometry.

Table 13: Summary of t-test analysis of SS2 students' achievement reflecting their misconceptions about the angle sum of a triangle

Groups	N	Mean (x)	Std Deviation	t-cal	t-crit	df	Inference
Treatment	104	6.16	2.76	18.50	1.55	208	Significant
Control	106	5.39	1.11				

Summary of Findings

1. Majority of students in the control (lecture) classroom were not able to develop correct concepts in various aspects of geometry investigated. They instead, developed a variety of misconceptions about those aspects of geometry.
2. A very small number of these control students were able to form the correct concepts of the aspects of geometry learnt in the study.
3. On the opposing side of the balance, majority of students in the treatment (experimental) classroom developed the correct concepts of the aspects of geometry learnt in the investigation.
4. Contrarily, only a small number of them had incorrect concepts of geometry.
5. That development of correct concepts in geometry (with respect to the treatment students), and incorrect geometry concepts (with respect to the control students) were attributable to classroom practices (exemplary practices with a variety of instructional materials and lecture with passivity) in the treatment and control classrooms respectively.
6. In terms of achievement, treatment students were superior to their control counterparts as evidenced by differences between their mean and standard deviation scores.
7. A significant effect of misconceptions about diagonals was spotted on students' achievement in geometry.

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8. Misconceptions about parallel and perpendicular lines significantly affected students' achievement in geometry.
9. Students' achievement in geometry was significantly affected by misconceptions about the general angle.
10. There was significant effect of students' misconceptions about the sum of angles in a triangle on their achievement in geometry.

Discussion

A lamentable discovery was made from this investigation that majority of students in the control classroom were not able to develop correct concepts about a number of aspects of geometry under investigation. This majority developed a variety of misconceptions about those aspects instead. For instance, 81 of the 106 control students developed misconceptions about diagonals; 83 of the 106 control had misconceptions about parallel and perpendicular lines; 86 of them had their misconceptions developed about the general angle; and 85 of them had incorrect concept of the sum of angles in a triangle. A very small number of these students (the control students) were able to form the correct concepts of the aspects of geometry learnt in this study. Only 25 of the 106 of these students managed to acquire the correct concepts of diagonals; just 23 out of 106 got the correct concept of parallel and perpendicular lines; only 20 got correct concept of the general angle; and a number as small as 21 out of 106 got the correct concept of the angle sum of a triangle. These are sad happenings as acquiring correct concepts in geometry is a pre-requisite for learning the subject. So when students are failing to develop correct geometric concepts, the geometry teacher becomes worrisome. Researchers elsewhere have reported similar findings in their investigations. For instance, French (2004) reported that majority of students in his study could not correctly state the number of sides and diagonals possessed by rectangles, triangles, etc. Also, Clements and Battista (2005), reported an investigation in which fewer than 10 students could find the measure of the third angle of a triangle, given the measures of the other two angles. Further, in another study, French (2004) stated that many students in secondary education held the misconception that the size of an angle is dependent upon the length of the two rays that form the angle.

A striking commendable discovery was also made regarding the development of correct geometric concepts in the treatment (experimental) classroom. In this classroom, majority of the students developed correct concepts of the aspects of geometry

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investigated by this study. It was discovered that 72 of the 104 treatment students developed correct concept of diagonals; 82 of these same 104 experimental students had correct concept of parallel and perpendicular lines; 80 of them acquired correct concept of the general angle; and 71 of the 104 developed correct concept of the angle sum of a triangle. Only a small number of these students developed incorrect concepts about the studied aspects of geometry. Just 32 of these 104 treatment students had misconceptions about diagonals; only 22 had misconceptions about parallel and perpendicular lines, merely 24 had incorrect concepts about the general angle; and 33 out of 104 had misconceptions about the sum of angles in a triangle.

Further, it was discovered that development of correct concepts (with respect to the treatment students), and incorrect geometry concepts (with respect to the control students), were attributable to classroom practices (exemplary practices with a variety of instructional materials), and (lecture with passivity) in the treatment and control classrooms respectively. In other words, the kind of teaching practice in the treatment class resulted to development of more correct concept and less incorrect concepts while that of the control class resulted to development of less correct concepts and more incorrect concepts.

Achievement wise, treatment students had superior achievement to that of the control students as evidenced by differences between their mean and standard deviation scores. Results in tables 6, 8, 10 and 12 indicated that differences in mean and standard deviation scores existed between the achievements of treatment and control students in favour of treatment students. Treatment students had higher mean and standard deviation scores than the control. Instructions in the treatment group were exemplary, involving the use of a variety of instructional materials by skilful and well experienced geometry teachers. Instructions in the control group on the other hand were delivered through the normal lectures with students mostly, passively listening. This explains why there were fewer misconceptions and more correct concepts developed in the treatment group while there were more misconceptions and less correct concepts developed in the control group. Achievement was therefore accordingly higher in the treatment class than it was in the control class.

A further discovery that was made was that misconceptions by students in the study area had significantly affected the learning of the various aspects of geometry studied. In particular, it was discovered that:

i) Misconceptions about diagonals had significant effect on students' achievement in geometry;

ii) Misconceptions about parallel and perpendicular lines had significant effect on students' achievement in geometry;

iii) Misconceptions about the general angle had significant effect on students' achievement in geometry; and

iv) Misconceptions about the sum of angles in a triangle had significant effect on students' achievement in geometry.

Conclusion

In this investigation, the following conclusion was reached:

1. Quite a greater number of students in the lecture classroom (control) than students in the experimental (treatment) class developed incorrect concepts of geometry
2. A very small number of these control students developed correct concepts of geometry
3. Majority of students in the treatment classroom developed correct concepts of geometry
4. Very few of control students had correct concepts of geometry
5. Development of correct or incorrect concepts by participants was attributable to the kind of teaching practice in use;
6. Achievement wise, the achievement of treatment students was superior to that of their control counterparts as evidenced by differences between their mean and standard deviation scores;
7. A significant effect of misconceptions about diagonals was spotted on students' achievement in geometry;
8. Misconceptions about parallel and perpendicular lines had significant effect on students' achievement in geometry;
9. Students' achievement in geometry was significantly affected by misconceptions about the general angle; and
10. There was a significant effect of students' misconceptions about the sum of angles in a triangle on students' achievement in geometry

Recommendations

Based on the findings of this investigation, the following recommendations were made:

1. Geometry teachers should de-emphasize classroom instructions through lecture during which students remain mostly passive and emphasize instructions which adopt practical approaches to teaching and use of instructional materials such that students remain active for most or all part of the lesson.
2. In the teaching of school geometry, teachers should ensure that students develop the correct concept of every aspect of geometry to be learnt through appropriate strategy. Correct concept development is a basic requirement in the learning of geometry
3. Geometry teachers should develop themselves by taking up programmes in faculties of education that acquaint them with global best practices in the teaching of geometry.
4. Geometry teachers are advised to patronize workshops, teachers' conferences and seminars, and other relevant means for further self-development on the job
5. Governments, proprietors, parents, philanthropists and other stake-holders in education should support teacher-education at all levels, especially for geometry/mathematics teachers, so that they become better teachers to handle geometry teaching properly.
6. Geometry teachers should embrace, and not resist, all the global best practices learnt or to be learnt as being recommended by this study.

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