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INVESTIGATING THE OPTIMALITY CRITERIA FOR A PARTIALLY BALANCED LATTICE DESIGN WITH TWO ASSOCIATE CLASSES

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ABSTRACT

Lattice designs is a class of incomplete block designs most commonly used in agricultural research. There is sufficient flexibility in the design to make its application simpler than most other incomplete block designs. The aim of this study is to investigate the design based optimality criteria of a two associate classes of a Partial Balance Lattice Design. The D, A, and E optimality criteria were studied. These approaches were demonstrated in our study involving

INTRODUCTION

Experimental design is usually characterized by size, number of experimental units, the procedure of allocation of treatments to the experimental units and the nature of blocking (Sir R. A.Fisher 1919-30). In some experiments it may not be possible to accommodate all the treatment combination in each block. Therefore incomplete block designs are preferable. If all the treatments are not used in each block of the plan of a design, then it is called an incomplete block design (IBD). Lattice designs are one class of incomplete block design most commonly used in agricultural research. There is sufficient flexibility in the design to make its application simpler than most other incomplete block designs. There are several types of lattice design the two of the most commonly used lattice designs are, the balanced lattices and the partially balanced lattice designs. Both require that the number of treatments must be a perfect square. In balanced lattices, the number of treatment is equal to the square of the number of units per block. The Partially Balanced Lattices

nine treatments. We investigated the robustness properties of each of these optimal designs using their relative efficiencies. The results show that D-optimal has the highest values of 27 and 729 respectively for one replicate, and two replicates designs, while A has 9, and 18, E has 3, 3, althrough the designs. Considering the efficiency of the designs, in maximizing the information matrix, the results show that D, has 3.57 while A has 40 and E have 50. In minimized the dispersion matrix, the results show that D has 96.498 while A has 40 and E have 50. The results above showed that partial lattice design with one and two possess D optimality criteria which maximize the information matrix. It is therefore recommended that an experimenter that chooses a Lattice design can do with a one or two replicated design without any loss of information.

Keywords: Lattice Square design, Associate classes, Optimality criteria, and Efficiency.

design is a subclass of incomplete block design is similar to the balanced lattice design but allows for a more flexible choice of the number of replications. While the partially balanced lattice design requires that the number of treatments must be a perfect square and that the block size is equal to the square root of this treatment number, the number of replication is not prescribed as a function of the number of treatments. In fact, any number of replications can be used in a partially balanced lattice design. They are well-known type of resolvable incomplete block designs.

Material And Method

The analysis was carried out using Matlap software. In this study we are considering m= 2 associate classes, and v = 9 treatments with r = 3 replicates arrange in b = 9 blocks. Considering a 3 X 3 triple lattice design. A relationship satisfying the following three conditions is called a partially balanced association scheme with m-associate classes. (i) Any two symbols are either first, second,…, or m^{th} associates and the relation of associations is symmetrical, i.e., if the treatment A is the i^{th} associate of treatment B, then B is also the i^{th} associate of treatment A. (ii) Each treatment A in the set has exactly n_i treatments in the set which are the i^{th} associate and the number n_i (i=1,2,…,m) does not depend on the treatment A. (iii) If any two treatment A and B are the i^{th} associates, then the number of treatments which are both j^{th} associate A and k^{th} associate of B is p^i_{jk} and is independent of the pair of i^{th} associates A and B. The number of v, n_1, n_2,…,n_m, p^i_{jk} (i,j,k

TIJSRAT
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=1,2,… ,m)are called the parameters of m-associate partially balanced scheme. Where r = number of replications, v = number of treatment in a block m = associate classes, n_1= number of treatment in first associates, n_2= number of treatment in second associates, λ_1 = First associate class, λ_1 = Second associate class,

Information and Dispersion Matrix

To use the later described criteria for the selection of the best design, we need to define two other types of matrices. The first is the information matrix $(X'X)$. This matrix is the multiplication of the tranpose of the design matrix X and X itself. The dispersion matrix $(X'X)^{-1}$ is the inverse matrix of this calculation (de Aguiar et al. 1995)

Data Analysis And Result

Table (1): Layout of the design

Blocks	Treatments		
1	1	2	3
2	4	5	6
3	7	8	9
4	1	4	7
5	2	5	8
6	3	6	9
7	1	6	8
8	2	4	9
9	3	5	7

Table (2): Show the 1st and 2nd associate and associates of all the treatments

Treatment number	First associates	Second associates
1	2, 3, 4, 6, 7, 8	5, 9
2	1, 3, 4, 5, 8, 9	6, 7
3	1, 2, 5. 6, 7, 9,	4, 8
4	5, 6, 1, 7, 2, 9	3, 4
5	4, 6, 2, 8, 3, 7	1, 9
6	4, 5, 3, 9, 1, 8	2, 7
7	8, 9, 1, 4, 3, 5	2, 6

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8	7, 9, 2, 5, 1, 6	3, 4
9	7, 8, 3, 6, 2, 4	1, 5

RESULT: (1)

From table (3) by inspection of the design we have constructed 2 associate classes with the following properties: number of treatment in first column (n_o) = 1, number of treatment in first associate (n_1) = 6, number of treatment in the second associate (n_2) = 2, first associate(λ_1) = 1, second associate $\lambda_2 = 0$, These associate classes assist us in the analysis of data from Lattice design experiment.

Design with one replication

Figure 1 show the matrix with 3 blocks, 9 treatments

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$The\ transpose\ of\ the\ matrix\ X'$

$$X' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$The\ information\ matrix\ is\ given\ by;$

$$(X'X) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$The\ dispersion\ matrix\ is\ given\ by$

$$(X'X)^{-1} = \begin{bmatrix} 0.3333 & 0 & 0 \\ 0 & 0.3333 & 0 \\ 0 & 0 & 0.3333 \end{bmatrix}$$

From the information matrix and dispersion matrix above,

$\boldsymbol{D - optimality\ is}$

$$max|X'X| \equiv min|X'X|^{-1} = 27\ or\quad 0.0370 \qquad \dots (1)$$

$A - optimality\ is$

$$Trace(X'X)^{-1} = \min[trace\ (X'X)^{-1}] = 9 \text{ or } 1 \quad \ldots (2)$$

$E - optimality\ is$

$$max\lambda_{min}(X'X) \equiv \min \lambda_{max}(X'X)^{-1} = 3\ or\ 0.3333 \quad \ldots (3)$$

Design With Two Replications

Figure 2 show the matrix with 6 blocks, 18 treatments

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$The\ transpose\ of\ the\ matrix\ X'$

$$X' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$The\ information\ matrix\ is\ given\ by;$

$$(X'X) = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

The dispersion matrix is given by

$$(X'X)^{-1} \begin{bmatrix} 0.3333 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3333 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3333 \end{bmatrix}$$

$\boldsymbol{D - optimality\ is}$

$$max|X'X| \equiv min|X'X|^{-1} = 729 \ \ or \quad 0.0014 \qquad \dots(4)$$

$\boldsymbol{A - optimality\ is}$

$$Trace(X'X)^{-1} = \min[trace\ (X'X)^{-1}] = 18 \text{ or } 3 \qquad \dots(5)$$

$\boldsymbol{E - optimality\ is}$

$$max\lambda_{min}(X'X) \equiv \min \lambda_{max}(X'X)^{-1} = 3\ or\ 0.3333 \qquad \dots(6)$$

DESIGN EFFICIENCY

$$Design\ efficiency = \frac{primary\ component\ (A)}{All\ compoenent\ s\ (A+B)} \times 100\%$$

Maximizing the information matrix

$$D - efficiency = \frac{729}{729+19683} \times 100 \qquad \dots(7)$$
$$= 0.0357 \ \times 100$$
$$= 3.57$$

Minimizing the dispersion matrix

$$D - efficiency = \frac{0.0014}{0.0014+5.0805e{-}05} \times 100 \qquad \dots(8)$$
$$= 0.9649 \ \times 100$$
$$= 96.498$$

Maximizing the information matrix

$$A - efficiency = \frac{18}{18+27} \times 100 \qquad \dots(9)$$
$$= 0.4 \ \times 100$$
$$= 40$$

Minimizing the dispersion matrix

$$A - efficiency = \frac{2}{2+3} \times 100 \quad \dots (10)$$
$$= 0.4 \times 100$$
$$= 40$$

Maximizing the information matrix

$$E - efficiency = \frac{3}{3+3} \times 100 \quad \dots (11)$$
$$= 0.5 \times 100$$
$$= 50$$

Minimizing the dispersion matrix

$$E - efficiency = \frac{0.3333}{0.3333+0.3333} \times 100 \quad \dots (12)$$
$$= 0.5 \times 100$$
$$= 50$$

Summary Of Results

Table (7): Rep 1 Maximizing the information matrix

Optimality criteria	D – optimality	A – optimality	E – optimality
Optimal	27	9	3
Efficiency	3.57	33.33	50

Table (8): Rep 1 Minimizing the dispersion matrix

Optimality criteria	D – optimality	A – optimality	E – optimality
Optimal	0.0370	1	0.3333
Efficiency	96.35	33.33	50

Table (9): Rep1 and Rep2 Maximizing the information matrix

Optimality criteria	D – optimality	A – optimality	E – optimality
Optimal	729	18	3
Efficiency	3.57	40	50

Table (10): Minimizing the dispersion matrix

Optimality criteria	D – optimality	A – optimality	E – optimality
Optimal	0.0014	2	0.3333

Efficiency	96.498	40	50

Discussion Of Result

In this research work, we have carefully constructed 2 associate classes of a Partially Balanced Lattices design of 9 treatments with three replications. In this manner, two treatments lying in the same row or in the same column of the association schemes are first associates meaning they occur together, while two treatments not lying in the same row or in the same column are second associates they do not occur together. We have demonstrated how good a design is maximized or minimized with respect to information matrix in one replicate and two replicated of Partial Lattices, Based on the analysis D-criteria has the highest value follow by A-criteria, then E-criteria. While in minimized the dispersion matrix, the result shows that D-criteria have the lowest value followed by E-criteria then A-criteria. Considering the efficiency of the design, the result of D, A and E efficiency which maximized the information matrix shows that D-criteria has a value 3.57, A criteria has the value 40 and E-criteria have the value of 50, while in minimized dispersion matrix the results shows that D-criteria has the value 96.498 while A criteria have 40 and E-criteria have 50.

Conclusion/Recommendation

In conclusion since in experimental design, a good criterion for one design may not be the best in another design. We have shown that the Partial Lattice possesses D optimality criteria which advices an experimenter to employ any of the designs. Considering the nature of our research and the result of the experiment which informs the experimenter to predict the result in advance, D-optimal is the best suited for Partially and Balanced Lattice design.

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