INTERNATIONAL JOUNAL OF: SCIENCE RESEARCH AND TECHNOLOGY

TIMBOU-AFRICA ACADEMIC PUBLICATIONS
INTERNATIONAL JOURNAL, AUGUST, 2021 EDITIONS
VOL. 6 NO. 9 ISSN: 2623-7861

COMPUTATIONAL ANALYSIS OF CHANGES IN THE INITIAL CONDITION ON DATA PRECISION OF TWO ECOLOGICAL SPECIES

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ABSTRACT

For the purpose of this study, a continuous dynamical system of ordinary differential equations of first order that are non-linear has been used to model the data precision of ecological species. Data precision is useful in selection of data for research purposes. Ordinary Differential Equation 45 (ODE45) numerical method has

INTRODUCTION

Competition is a type of interaction in which species struggle for limited resources available. According to Keddy & Cahill (2018), competition refers to the negative effects on plant growth or fitness caused by the presence of neighbours, usually by reducing the availability of resources. Whilst definitions of competition abound, they can typically be divided into two categories, those that focus on mechanisms and resource acquisition and those that focus on the reduction in fitness brought about by a shared requirement for a resource in limited supply (Silvertown & Charlesworth, 2009). Donald (1951) opined that competition is evident in dense populations shortly after germination and thereafter becomes operative progressively in populations of lower density.

The Lotka-Volterra model was developed to enable ecologists predict the potential outcome when two species are in competition for the same resources. It describes the outcome of competition between two species over ecological time. Basically, the model attempts to account for the effect that the presence of one species will have on the population growth of the other species, relative to the competitive effect that two members of the same species would have on each other. A differential equation is an equation that involves one or more terms and the derivative of dependent variable with respect to the independent variable. There are different types of differential equations namely first order, partial, non linear, homogenous, non homogenous, ordinary, linear, just to mention a few. A non linear ordinary differential equation do not have closed form solutions hence the use of mathematical softwares like Matrix Laboratory (MATLAB), Mathematica, Maple, Maxima, MathCad to solve it. Since a non-linear system does not have a closed form solution, the interacting functions are coded in a MATLAB computer programming language in order to study the behavior of the system (George 2018, Ekaka-a 2009).

A mathematical model is not an exact quantification of a real life situation and model parameter values are not chosen using any probabilistic law so they are prone to errors hence the need for data precision to get best fit data and to minimize errors .The concept of data precision that is derived from the statistical

been applied to show the effect of decreased variations of the initial condition on data precision. The study creates awareness on the application of data precision in ecology and enlightens experts on the effects of varying initial condition parameter values on data precision of interacting ecological species. It was observed that 5% variation of the initial value conditions together gave a very low data precision value for y- data set which indicates best fit data. The results also show that y-data set is a better data set compared to x-data set and can be used for further research studies.

KEYWORDS: *Initial, Condition, Computational Analysis, Effect, Data precision, Ecological species.*

theory of standard deviation is as old as the subject of statistical sciences. It is against this background that we can find several of its application in several areas of knowledge like engineering, agriculture, management science, education, medical science, aerospace, geosciences just to mention a few.

Data precision is very important in ecological processes because if the right model parameters are not chosen, invalid results will be got which cannot be used for further research studies. Data precision deals with the calculation of standard deviation and the principle states that the smaller the value of the standard deviation of a data set, the better the data set.

Data precision is easy to calculate for data sets with a small sample size but it becomes more challenging when the sample size is large. To tackle this numerical mathematical problem which will be time consuming to solve analytically, a computational method that is faster and more efficient will be used. The method of using mathematical codes with the help of Matrix Laboratory (Matlab) is adopted for this research study of two ecological species namely beans and maize.

Bella (1971) worked on a new competition model for individual trees using numerical simulation. The result showed that a big subject tree receives competition from its immediate neighbours whereas a small subject tree may be affected by bigger competitors from a considerable distance.

Ahmad & Lazer (2005) studied the average growth and extinction in competitive Lotka Volterra systems. A non linear model was used and the analysis showed that for the two species autonomous competitive Lotka Volterra model with no fixed point in the open positive quadrant, one of the species is driven to extinction, while the other population stabilizes at its own carrying capacities.

Cushing (1986) studied the classic Lotka Volterra equation for two competing species with constant coefficients and the stability of periodic solutions is related to the theory of competition exclusion. It was shown that two species which could not coexist in a constant environment can coexist in a limit cycle fashion when subjected to suitable harvesting or removal rate.

Leibold & Mcpeek (2006) explored the concept that species are equivalent to each other in all important ecological communities, how much species may arise evolutionarily, and how the possibility of ecological equivalents relates to previous ideas about niche differentiation using the neutral or niche model. It was shown that the co-occurrence of ecologically similar or equivalent species is not compatible with niche theory, because niche relation can sometimes favour coexistence of similar species.
Svanback & Bolnick (2006) studied population density of three spine sticklebacks in enclosures in a natural lake. They observed that increased population density led to reduced prey availability, causing individuals to add alternative prey types to their diet. Competition also increased the diet morphology correlations, so that the frequency dependent interactions were stronger in high competition.
Shukla *et al.* (2001) investigated the survival of two competing species in a polluted environment by using the method of local stability analysis. They showed that competitive outcomes may be affected in the presence of a toxicant.

Mathematical Equations

By using the theory of modified Lotka-Volterra competition model and following Ekaka-a (2009), the following continuous dynamical system of non linear first order differential equations describing the interaction between two species will be considered.

$$\frac{dx(t)}{dt} = \alpha_1 x - \beta_1 x^2 - r_1 xy$$

$$\frac{dy(t)}{dt} = \alpha_2 y - \beta_2 y^2 - r_2 xy$$

With $x\,(0) = 0.45$ and $\mathrm{y}\,(0) = 0.68$

Where $x(t)$ and $y(t)$ represent the plant biomass of the first and second plant species respectively at time t.
α_1 and α_2 are called the intrinsic growth rates for populations $x(t)$ and $y(t)$ in the absence of self-interaction and inter-competition interaction.
β_1 and β_2 represent the intra-competition coefficients.

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ISSN: 2623-7861

r_1 and r_2 represent the inter-competition coefficients.
To analyze the proposed problem, the following model parameter values have been assumed.
$\alpha_1 = 0.168$, $\beta_1 = 0.0020339$, $r_1 = 0.0018$, $\alpha_2 = 0.002$, $\beta_2 = 0.001$, $r_2 = 0.0006$

Method of Analysis

For the analysis of this present study, the procedures have been described in the following steps:

i) The proposed systems of nonlinear ordinary differential equations were coded and simulated using ODE 45 numerical scheme.
ii) The initial conditions $x(0) = 0.45$ and $y(0) = 0.68$ were then varied at 5%, 10%, 15% and 20%.
iii) The region of high data precision is an indication of poor data selection whereas the region of low data precision is an indication of best fit data selection.

Results

Table 1.1: ODE45 numerical calculation of data precision when all model parameters are fixed (variation at 100%)

x(t)	y(t)
0.450000000000000	0.680000000000000
0.531140960175327	0.680698020440773
0.626800048794635	0.681360232310412
0.739529029348397	0.681980111024090
0.872315082068679	0.682549979753590
1.028639500595697	0.683060839093813
1.212559578058866	0.683502130820001
1.428782180397256	0.683861510257248
1.682757983179862	0.684124554836392
1.980768652428369	0.684274466043080
2.330024459750099	0.684291727787944
2.738755017676432	0.684153735897391
3.216290465406719	0.683834427396244
3.773130151681513	0.683303859548077
4.420969401400906	0.682527861684860

5.172696256715114	0.681467630996278
6.042299168280049	0.680079510855833
7.044711542685844	0.678314741655513
8.195505766241029	0.676119546714003
9.510479328455917	0.673435275806979

Table 1.1.1: Statistical values: range, mean, variance, standard deviation of x and y data sets

Range	Mean	Variance	Standard deviation
9.060479328455918	3.149907728667036	7.507228763356785	2.739932255249532
0.010856451980966	0.681647008146126	0.000008482665773	0.002912501634916

Table 1.2: Computational calculation of data precision due to a 5% variation of the initial conditions together using ODE 45 numerical method

x(t)	y(t)
0.022500000000000	0.034000000000000
0.026613117289801	0.034066407859078
0.031477874302446	0.034132851436054
0.037231416752151	0.034199313652469
0.044036095662593	0.034265774203473
0.052083591645949	0.034332209018965
0.061600698939714	0.034398589512263
0.072855261650094	0.034464881800259
0.086163909778298	0.034531045691563
0.101900687064680	0.034597033546678
0.120507312447669	0.034662788924571
0.142505663467870	0.034728244922714
0.168511343333812	0.034793322369216
0.199251549781523	0.034857927427780
0.235583014989269	0.034921949176232
0.278517201605409	0.034985256195096
0.329244056875530	0.035047693308337
0.389166893722994	0.035109076797527

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0.459933832440183	0.035169189994397
0.543484504965199	0.035227776751643

Table 1.2.1: Statistical values: range, mean, variance, standard deviation of x and y data sets

Range	Mean	Variance	Standard deviation
0.520984504965199	0.170158401335759	0.024149561600081	0.155401292144182
0.001227776751643	0.034624566629416	0.000000148024490	0.000384739509693

Table 1.3: Computational calculation of data precision due to a 10% variation of the initial conditions together using ODE 45 numerical method

x(t)	y(t)
0.045000000000000	0.068000000000000
0.053220322546911	0.068129495732390
0.062941184110028	0.068258862683203
0.074435844100777	0.068388032213839
0.088027594130754	0.068516922854737
0.104097871317011	0.068645438175303
0.123097632360001	0.068773463814945
0.145558962264641	0.068900864412025
0.172110188438952	0.069027479622090
0.203492619341383	0.069153119651701
0.240580403598904	0.069277559964256
0.284404486685574	0.069400534819965
0.336178503772530	0.069521730264772
0.397332559101065	0.069640774896115
0.469546763101139	0.069757230589015
0.554797866626634	0.069870579511848
0.655402314055536	0.069980211868581
0.774078879321234	0.070085408010430
0.914003047979168	0.070185322292111
1.078887165923527	0.070278959298115

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Table 1.3.1: Statistical values: range, mean, variance, standard deviation of x and y data sets

Range	Mean	Variance	Standard deviation
1.033887165923527	0.338859710438788	0.095256666801526	0.308636787829199
0.002278959298115	0.069189599533772	0.000000517802796	0.000719585155826

Table 1.4: Computational calculation of data precision due to a 15% variation of the initial conditions together using ODE 45 numerical method

x(t)	y(t)
0.067500000000000	0.102000000000000
0.079821617441062	0.102189264200007
0.094389938061697	0.102378036237995
0.111613307333058	0.102566161763625
0.131974554278879	0.102753457725690
0.156042960292370	0.102939707636113
0.184491032067417	0.103124654970925
0.218111523622007	0.103307996368147
0.257839577532217	0.103489372814906
0.304777069479348	0.103668359805460
0.360221418877076	0.103844455694405
0.425700040668985	0.104017067559250
0.503007367593179	0.104185495911819
0.594252665432060	0.104348914681849
0.701906940395319	0.104506351253220
0.828867471224113	0.104656658692407
0.978516007776488	0.104798489859236
1.154802596085890	0.104930259865867
1.362315189978756	0.105050112934427
1.606381404674952	0.105155873885324

Table 1.4.1: Statistical values: range, mean, variance, standard deviation of x and y data sets

Range	Mean	Variance	Standard deviation
1.538881404674952	0.506126634140744	0.211369036547240	0.459748884226205
0.003155873885324	0.103695534593034	0.000001011333087	0.001005650579003

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ISSN: 2623-7861

Table 1.5: Computational calculation of data precision due to a 20% variation of the initial conditions together using ODE 45 numerical method

x(t)	y(t)
0.090000000000000	0.136000000000000
0.106417003641382	0.136245713841851
0.125824144789855	0.136490374595613
0.148763831705812	0.136733708376089
0.175877034881409	0.136975390573837
0.207918979577297	0.137215037535677
0.245781129198075	0.137452194976165
0.290513365975750	0.137686326074684
0.343352815317669	0.137916796064999
0.405755303276359	0.138142855101593
0.479432489833281	0.138363618018728
0.566395868870021	0.138578039892121
0.669003767160629	0.138784889706983
0.790021401844949	0.138982716094670
0.932679042572688	0.139169813403838
1.100751108999354	0.139344174815504
1.298625647591945	0.139503449215874
1.531403306359044	0.139644879071982
1.804975200404768	0.139765246891079
2.126135735539287	0.139860797335603

Table 1.5.1: Statistical values: range, mean, variance, standard deviation of x and y data sets

Range	Mean	Variance	Standard deviation
2.036135735539288	0.671981358876979	0.370611690577837	0.608778851946942
0.003860797335602	0.138142801079345	0.000001547999647	0.001244186339515

Discussion of Result

NOTE: For the second tables, the first row is for x data set while the second row is for y data set.

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ISSN: 2623-7861

For Table 1.1, two classified time dependent data which are parallel data sets have been found. The data set x(t) has the smallest value of 0.450000000000000 and the biggest value of 9.510479328455917 thereby producing the range of 9.0605 approximately whereas the data set y(t) has the smallest value of 0.673435275806979 and with the biggest value 0.684291727787944 producing the range of 0.0109 approximately. By comparing these data sets, it has been observed that x(t) data set spreads faster than the data set y(t). Another observation concerns the fact that the data set y(t) is also associated with a value of standard deviation having its value of 0.0029 approximately which is smaller than the standard deviation of the data set x(t) which is 2.7399 approximately provided all model parameters are fixed.

For Table 1.2, two classified time dependent data which are parallel data sets have been found. The data set x(t) has the smallest value of 0.022500000000000 and the biggest value of 0.543484504965199 thereby producing the range of 0.5210 approximately whereas the data set y(t) has the smallest value of 0.034000000000000 and with the biggest value 0.035227776751643 producing the range of 0.0012 approximately. By comparing these data sets, it has been observed that x(t) data set spreads faster than the data set y(t). Another observation concerns the fact that the data set y(t) is also associated with a value of standard deviation having its value of 0.0004 approximately which is smaller than the standard deviation of the data set x(t) which is 0.1554 approximately provided the initial value conditions are varied together by 5%.

For Table 1.3, two classified time dependent data which are parallel data sets have been found. The data set x(t) has the smallest value of 0.045000000000000 and the biggest value of 1.078887165923527 thereby producing the range of 1.0339 approximately whereas the data set y(t) has the smallest value of 0.068000000000000 and with the biggest value 0.070278959298115 producing the range of 0.0023 approximately. By comparing these data sets, it has been observed that x(t) data set spreads faster than the data set y(t). Another observation concerns the fact that the data set y(t) is also associated with a value of standard deviation having its value of 0.0007 approximately which is smaller than the standard deviation of the data set x(t) which is 0.3086 approximately provided the initial value conditions are varied together by 10%.

For Table 1.4, two classified time dependent data which are parallel data sets have been found. The data set x(t) has the smallest value of 0.067500000000000 and the biggest value of 1.606381404674952 thereby producing the range of 1.5389 approximately whereas the data set y(t) has the smallest value of 0.102000000000000 and with the biggest value 0.105155873885324producing the range of 0.0032 approximately. By comparing these data sets, it has been observed that x(t) data set spreads faster than the data set y(t). Another observation concerns the fact that the data set y(t) is also associated with a value of standard deviation having its value of 0.0010 approximately which is smaller than the standard deviation of the data set x(t) which is 0.4597 approximately provided the initial value conditions are varied together by 15%.

For Table 1.5, two classified time dependent data which are parallel data sets have been found. The data set x(t) has the smallest value of 0.090000000000000 and the biggest value of 2.126135735539287 thereby producing the range of 2.0361 approximately whereas the data set y(t) has the smallest value of 0.136000000000000 and with the biggest value 0.139860797335603 producing the range of 0.0039 approximately. By comparing these data sets, it has been observed that x(t) data set spreads faster than the data set y(t). Another observation concerns the fact that the data set y(t) is also associated with a value of standard deviation having its value of 0.0012 approximately which is smaller than the standard deviation of the data set x(t) which is 0.6088 approximately provided the initial value conditions are varied together by 20%.

Conclusion and Recommendation

With the use of an ODE 45 numerical method, a continuous dynamical system of ordinary differential equations of first order that are non-linear has been used to model the data precision of ecological species. It was observed that 5% variation of the initial value conditions together gave a

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ISSN: 2623-7861

very low data precision value for y- data set which indicates best fit data. The results also show that y-data set is a better data set compared to x-data set and can be used for further research studies.

The region of small data precision is an indication of best fit data selection which can be used to revalidate the original model formulations for the purpose of further studies. This present study can thus be extended to evaluate the effect of varying the initial value conditions in combination with intrinsic growth rate parameters on the data precision of ecological species.

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