HP
JECM 2020
Hummingbird Publications
Journal of Env. Design & Constructions Mgt.
www.hummingpubng.com
© June, 2020
Vol. 20 No. 4

Development of Geodetic Vertical Control Network Computational Algorithms for Affective and Sustainable Environment

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Keyword:
Computational algorithm, Vertical control point, Bench mark, Adjustment and Model

Abstract
This research is aimed at developing geodetic vertical control network computational algorithms for affective and sustainable environment. The objectives of this research include; establishment of geodetic vertical control stations, formation of matrices and development of computational algorithms. Two bench marks were used as reference datum for connection. The reduced level of the bench mark was collected from the Department of Surveying and Geo-informatics, Federal Polytechnic, Bauchi as shown in Table 1. Eight (8) geodetic vertical control stations were established and their respective coordinate were determined using Global Positioning System (GPS) receiver. The geodetic vertical control network is shown in Figure 1. The horizontal and vertical controls are recorded in Table 2. The height difference was deduced from the observed reduced level which was used in the formation of the matrices. The computational algorithms were developed for parametric and condition equation model as shown in Figure 2 and Figure 3 respectively. The results

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generated from these algorithms indicated that, it is the fastest and less rigorous procedure of least square adjustment of large geodetic vertical control network. The result obtained using these computational algorithms are shown in Table 4. It is recommended that surveyors/engineers should be conversant with the procedure of using programmable calculators, android phones and computers in order to facilitate precise and accurate results, when discharging their responsibilities for effective management of environment.

Introduction

Background

A geodetic control network is the wire-frame or the skeleton on which continuous and consistent mapping, Geographic Information Systems (GIS), and surveys are based. Traditionally, geodetic control points are established as permanent physical monuments placed on the ground and precisely marked, located, and documented (Okwuashi, 2014). Vertical control points are survey points which are described and monumented (or otherwise permanently marked) and whose elevations are determined in an adjustment or whose elevations are available from other sources (Moritz 1978). A bench mark is a point of reference used in heights determination. Dermanis (1992) opined that least square adjustment of heights usually generated best results which are used in building, civil engineering work and environmental monitoring etc.

During the building of large structures, such as dams, bridges and buildings, it is useful to lay out various geodetic vertical control points in predetermined locations. The availability of control points is naturally desirable. As well, it is often necessary to know the a priori and a posteriori movements of the ground and water levels. In the case of dams, water tunnels and irrigation constructions, the exact knowledge of the equipotential surfaces are also needed (Ayeni, 2001). “Least Square Adjustment” is recommended as the best method for adjusting various surveying observations and computations used in map

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making. Its mathematical complexity and analysis remains however, a mystery to some students of Surveying (Ayeni, 2001).

The major problem that prompted this research is that there is an inadequate precise and accurate geodetic vertical control points within the study area. The limited bench marks established was not properly adjusted for gross error. This is either because the adjustment procedures involved are rigorous or the then surveyors are ignorant of the application of least square adjustment.

Least squares estimation is a regression technique based on minimization of residuals, has been invaluable in bringing the best fit solutions to parameters in science, environmental and engineering. However, in dynamic environments such as in Geomatics engineering, formation of these equations as well as using the model can be very challenging. To overcome these challenges, the derivation of the models should be well understood and interpreted in such a way that computer or scientific calculator such as Casio Fx9750G plus power graphic can be applied. With the application of this hardware a program can be design to solve a very large and comprehensive network.

This research uses observation and condition equation model to developed geodetic vertical network computational algorithms. The method of observation equations may be defined as equations in which each adjusted observation is expressed as a function of some unknown adjusted parameters (Paul, 2002). Such a functional relationship will be valid only if the observations and the unknown parameters assume their "true" values (i.e values without errors). Since in practice the true values of these quantities are not obtainable instead we make use of the most probable or adjusted observations and adjusted unknown parameters. The word "adjusted" is used to indicate that their calculated or observed values have been adjusted for the errors associated with them (Ayeni, 2001). The functional relationship between each adjusted observation and the adjusted parameters may be represented mathematically as follows;

$L^a = f(X^a)$

where L^a = adjusted observation,

X^a = adjusted parameters,

L^b = original observations,

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V = Vector of residuals,

$L^a = L^b + V$

The model to be adopted for the development of the computational algorithm for parametric method is derived as follows;

$L^a = F(X^a)$__________ (1)

Where L^a = Adjusted observation

X^a = Adjusted parameters

L^a = Original Observations

$\hat{x}$= Approximate value to the observation

$L^a = L^b + V$ and $X^a = \hat{x}$ ________(2)

Substitute equation (2) into equation (1)

$L^b + V = F(\hat{x})$ by partial differentiation

$L^b + V = \partial f(\hat{x})\ X$

$X^o = 0$ in linear model

Where $\partial f(\hat{x}) = A$

$L^b + V = Ax$

$V = AX - L$ ___ (3) Normal equation

The purpose of least square problem is to obtain the estimate $\hat{x}$ of X by minimizing the sum of the square of the weighted residuals V^TPV known as Lagrange multiplier, where P is the apriori weight of observations.

This means that V^TPV = Minimum ______ (4)

Now substituting eqn. 3 into eqn. 4,

We have,

$F = (AX - L)^T P (Ax - L)$ = Minimum.______(5)

However we shall take partial derivation of eqn. 6 and equate them to zero

$F = (Ax - L)^T P (AX - L) = O$ ___________(6)

Expanding equation 6 we have,

$F = A^TX^TAX - A^TX^TPL - AXL^T P - L^T PL = 0$ ___(7)

Differentiating F w.r.t X^T

$$\frac{\partial f}{\partial f X^T} = A^T PAX - A^TPL$$

$$A^T PAX = A^TPL$$

$X = (A^T PA)^{-1} A^TPL$ model.__(8)

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this is the parametric model

Condition Equation Method

The method of condition equations otherwise known as the method of correlates establishes a set of equations which must be satisfied by the true values of observations, given certain geometrical conditions or physical laws of nature imposed by the configuration of the problem (Ayeni, 2001). The model to be adopted for the development of the computational algorithm for condition equation method is derived as follows;

$F (L^a) = 0$ ________(9)

$L^a= L^b + V$ ________(10)

Where L^a, L^b and V were defined initially.

$F (L^a) = F (L^b + V)$_________(11)

Taking the partial deviation, we have

$$= \partial f (L_b) + \frac{\partial f(L^b) V}{\partial f (L^a)}$$

Where $\partial f (L^b) = w$ and $\frac{\partial f (L^b)}{\partial f (L^a)} = B$. substituting

W and B

W+BV or BV +W = _______ (12) Normal equation.

Introducing the Lagrange multipliers as vector correlate $V^TPV -2K^T$

$F = V^TPV - 2K^T (W +BV)$ _______ (13)

$= V^TPV -2K^TW - 2K^TBV$ _______(14)

Partial differentiation of F w.r.t $V_1$and K^T

$\frac{\partial f}{\partial V} = PV – B^TK$ ___ (6) and $\frac{\partial f}{\partial K^T} = W+ BV$

____(15)

$V= P^{-1}B^TK$ __________(16)

Putting equation 16 into eqn.15, we have

$W + B(P^{-1} B^TK) = W + BP^{-1} B^TK = 0$

$K = (BP^{-1}B^T)^{-1}W$ ___________ (17)

Putting equation 9 into equation (8)

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$V = - P^{-1} B^T (BP^{-1}B^T)^{-1}W$ ___(18) this is the model.

Where $M=BP^{-1} B^T$ and

$V = - P^{-1} B^T M^{-1} W$, ___(19)

Study Area

The study area is Bauchi metropolis located in Bauchi state, with maximum and minimum temperature of about 41°c at day time and 23°c at night. Bauchi state is located between latitudes 09^0 30' N and 09^0 50' N, north of the equator and longitudes 09^0 50' E and 10^0 20' E, east of the Greenwich meridian.

Materials and Methods

The hardware and the software used includes; GPS receiver, AutoCAD 2010 version, Casio Fx 9750G Plus power graphic and computer. Eight (8) geodetic vertical control stations were monumented to satisfy the criteria for both parametric and condition equation model. GPS receiver was the equipment adopted for the acquisition of the horizontal and vertical controls. The horizontal controls were used for designing the spatial position of the bench mark and the distances between the stations were also derived. The reduced level of the bench mark was used in computing the height difference. The matrix of A, P, L and the matrix of B, P, W were derived from the computed height difference for observation and condition equation model respectively. Casio Fx 9750G plus power graphic was used for designing the computational algorithms. The apriori data stored in the power graphic are the matrix of A, P, L and B, P, W for observation and condition equation model respectively. The flowcharts of the design of the computational algorithms were developed for parametric and condition equation model as shown in Figure 2 and Figure 3 respectively. The picture of Casio Fx 9750G plus power graphic containing the designed computational algorithm is as shown in Figure 4.

Table 1: Bench Mark used for Connection

Easting	*Northing*	*Reduced Level*	*Bench Mark*
590630.023	1147948.555	617.904	BM 003
586130.839	1133011.250	664.995	BM005

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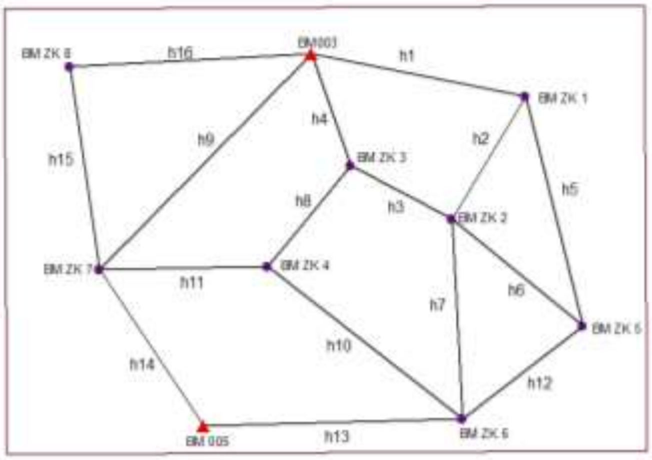
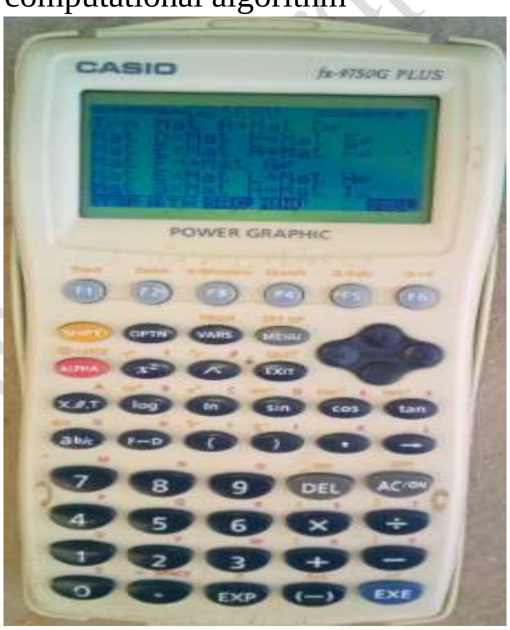


Figure 1: Geodetic Vertical Control Network (Source: Researcher's Lab)

Table 2: Horizontal and Vertical Control points

Easting	Northing	Reduced Level	Bench Mark
599425.025	1146303.516	606.131	BM ZK 1
596369.082	1141382.256	633.755	BM ZK 2
592201.886	1143525.385	653.743	BM ZK 3
588788.754	1139358.190	593.169	BM ZK 4
601845.967	1136937.247	598.750	BM ZK 5
596885.020	1133206.615	584.199	BM ZK 6
581764.052	1139278.514	693.362	BM ZK 7
580533.737	1147454.456	644.101	BM ZK 8

Figure 4: Casio Fx 9750G plus power graphic containing the designed computational algorithm



Results

The computational algorithm in Figure 2 was developed for parametric model which was used in the adjustment of the bench mark. The matrix of A, P and L was designed and used in the adjustment. The computational algorithm in Figure 3 was developed for condition equation model which was used in the adjustment of the bench mark. In this method, the designed matrix is B, P, and W. The final result of the adjustment is presented in Table 4.

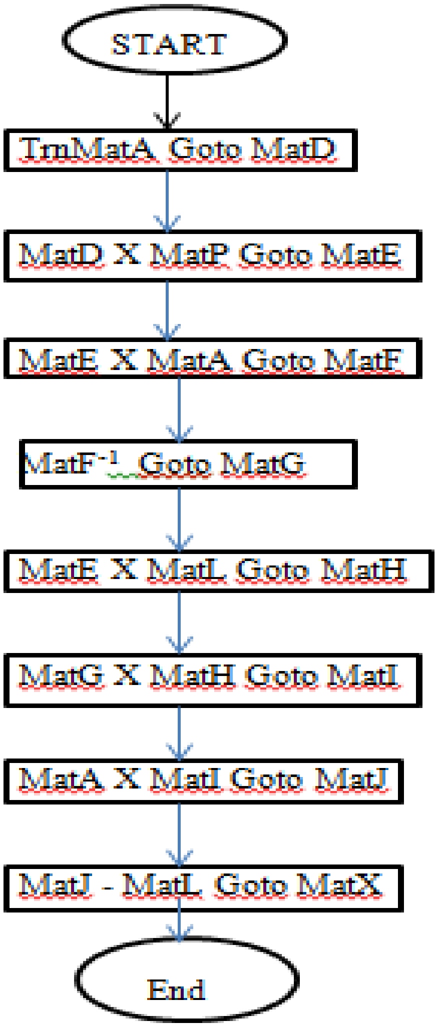


Figure 2: Computational Algorithm for Parametric Method (Source: Researcher's Lab)

In parametric method, the first approach is to compute for the approximate reduced level of all the points in the network. The computed approximate reduced level is as follows;

$BM\ ZK_1^0 = BM_{003} - h_1 = 606.131$ ________20

$BM\ ZK_2^0 = BM\ ZK_1 + h_2 = 633.755$______21

$BM\ ZK_3^0 = BM_{003} + h_4 = 653.743$ _______22

$BM\ ZK_4^0 = BM\ ZK_3 - h_8 = 593.169$ _____23

$BM\ ZK_5^0 = BM\ ZK_1 - h_5 = 598.750$_____24

$BM\ ZK_6^0 = BM_{005} - h_{13} = 584.199$______25

$BM\ ZK_7^0 = BM_{005} + h_{14} = 693.362$ _____25

$BM\ ZK_8^0 = BM_{003} + h_{16} = 644.101$_____26

The height difference h_1, h_2 ……h_{16} were computed in order to deduce for the matrix of L as shown in the fourth column of Table 3.

$h_1 = BM_{003} - BMZK_1 = 617.904 - 606.131 = 11.773$ __27

$h_2 = BM\ ZK_2 - BMZK_1 = 633.755 - 606.131 = 27.624$__28

$h_3 = BM\ ZK_3 - BMZK_2 = 653.743 - 633.755 = 19.988$__29

$h_4 = BM\ ZK_3 - BM_{003} = 653.743 - 617.904 = 35.839$__30

$h_5 = BM\ ZK_1 - BMZK_5 = 606.131 - 98.750 = 7.381$___31

$h_6 = BM\ ZK_2 - BMZK_5 = 633.755 - 98.750 = 35.005$__32

$h_7 = BM\ ZK_2 - BMZK_6 = 633.755 - 84.199 = 49.556$__33

$h_8 = BM\ ZK_3 - BMZK_4 = 653.743 - 3.169 = 60.574$__34

$h_9 = BM\ ZK_7 - BM_{003} = 693.362 - 617.904 = 75.458$__35

$h_{10} = BM\ ZK_4 - BMZK_6 = 593.169 - 84.199 = 8.970$__36

$h_{11} = BM\ ZK_7 - BMZK_4 = 693.362 - 3.169 = 100.193$_37

$h_{12} = BM\ ZK_5 - BMZK_6 = 598.750 - 84.199 = 14.551$_38

h_{13}=BM $_{005}$–BMZK $_{6}$=664.995-584.199=33.763__39
h_{14}=BM ZK$_{7}$–BM $_{005}$=693.362-664.995=28.367__40
h_{15}=BM ZK$_{7}$–BMZK$_{8}$=693.362-44.101=49.261__41
h_{16} =BMZK$_{8}$-BM$_{003}$=644.101 –617.904=26.197__42

Table 3: Height Difference

Line	*Observed Height diff.*	*Computed Height diff.*	*Element of L matrix*	*Dist (km)*
h_1	11.767	11.773	0.006	9
h_2	26.875	27.624	0.749	6
h_3	20.743	19.988	-0.755	5
h_4	35.851	35.839	-0.012	5
h_5	7.387	7.381	-0.006	10
h_6	34.262	35.005	-0.743	7
h_7	48.821	49.556	0.735	8
h_8	60.586	60.574	-0.012	5
h_9	75.460	75.458	-0.002	12
h_{10}	8.978	8.970	-0.008	10
h_{11}	100.195	100.193	-0.002	7
h_{12}	14.559	14.551	-0.008	6
h_{13}	33.713	33.763	-0.050	11
h_{14}	28.369	28.367	-0.002	8
h_{15}	49.263	49.261	-0.002	8
h_{16}	26.198	26.197	-0.001	10

$$A=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$L = \begin{bmatrix} 0.006 \\ 0.749 \\ -0.755 \\ -0.012 \\ -0.006 \\ -0.743 \\ 0.735 \\ -0.012 \\ -0.002 \\ -0.008 \\ -0.002 \\ -0.008 \\ 0.050 \\ -0.002 \\ -0.002 \\ 0.001 \end{bmatrix}$$

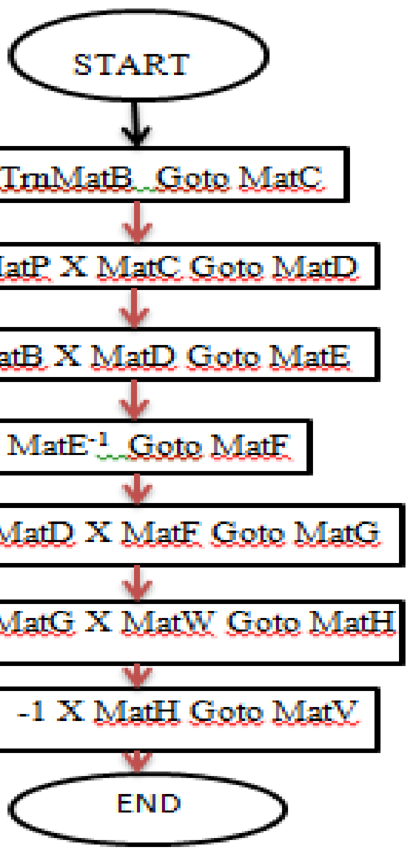

Figure 3: Computational Algorithm for Condition Method (Source: Researcher's Lab)

The condition equations are as follows;

$-(h_1 + V_1) + (h_2 + V_2) + (h_3 + V_3) - (h_4 + V_4) = 0$ __43

$-(h_1 + V_1) + (h_3 + V_3) - (h_4 + V_4) - (h_5 + V_5) + (h_6 + V_6) = 0$ __44

$-(h_1 + V_1) - (h_5 + V_5) - (h_{12} + V_{12}) + (h_{13} + V_{13}) = 0$__45

$(h_4 + V_4) - (h_3 + V_3) - (h_7 + V_7) + (h_{13} + V_{13}) = 0$__46

$(h_4 + V_4) - (h_8 + V_8) - + (h_{13} + V_{13}) = 0$____47

$(h_9 + V_9) - (h_{10} + V_{10}) - (h_{11} + V_{11}) + (h_{13} + V_{13}) = 0$___48

$(h_9 + V_9) - (h_{114} + V_{14}) = 0$____49

$-(h_{16} + V_{16}) + (h_{15} + V_{15}) - (h_{14} + V_{14})$ ____50

$$
B = \begin{bmatrix}
-1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1
\end{bmatrix}
$$

$$
W = \begin{bmatrix} 0 \\ 0.003 \\ 0 \\ 0.001 \\ -0.001 \\ 0 \\ 0.002 \\ 0.08 \end{bmatrix}
$$

$$
P = \begin{bmatrix}
0.11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.14 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.17 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.09 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.13 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.13 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1
\end{bmatrix}
$$

The adjusted reduced levels of the bench mark computed are shown in Table 4.

Bench mark	*parametric model*	*condition equation model*
BM ZK 1	606.145	606.145
BM ZK 2	633.013	633.014
BM ZK 3	653.756	653.757
BM ZK 4	593.167	593.167
BM ZK 5	598.751	598.751
BM ZK 6	584.190	584.192
BM ZK 7	693.365	693.365
BM ZK 8	644.103	644.102

Table 4: Result of the Adjustment

Conclusion

It is concluded that the aim of this research was achieved and the developed computational algorithms is functional in solving all geodetic/surveying application that would involve the use of least square adjustment model.

Recommendation

It is recommended that precise and accurate geodetic vertical controls should be distributed randomly within Bauchi Metropolis for engineering, geodetic and environmental purposes. This will go a long way in improving environmental sustainability.

It is also recommended that surveyors and engineers should be conversant with the procedure of using programmable calculators, android phones and computers in order to facilitate precise and accurate results, when discharging their responsibilities for effective management of environment.

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