MATHEMATICAL MODELLING OF THE TRANSPORTATION OF PETROLEUM PRODUCTS FROM SOURCE TO DESTINATION

EMEKA MICHAEL OKOYE ¹, MADU IFY ², SANI USMAN ³

¹ ICT Dept, Wellspring University Benin City Edo State
², ³ Computer Science Dept, Fed. Poly Bauchi

ABSTRACT

The proposed Mathematical model of transporting petroleum products from the source to various destinations is considered in this research. The data gathered were modeled using the modified distribution method and the North West Corner Rule, representing the transportation problem as a tableau and solving it with the computer software solver to generate a minimized transportation cost by finding the shortest distance. This Mathematical model will be useful for making strategic decisions by the logistics managers of Pipeline and Products Marketing Companies, it will also be useful in making optimum allocation of the production from the source to several destinations at a minimum transportation cost.

INTRODUCTION

The problem under study is the urgent transportation of petroleum products from the refinery to various filling stations and sub-depot scattered all over the region. (Adlakha.V, et.al 2006) The process of finding the transport scheme, will involve considering the shortest distance, and minimizing the total cost of transportation.

Since customers are scattered in different locations all over the region, unless there is an efficient logistics system, logistics cost would be respectively high. (Brandao et.al 2006) In the past, when the operation of picking up petroleum products took place, the only thing that was used to assist planning truck routes was the experience of the truck driver. The truck driver was the one who decided his truck route. Many times, there was back and forth movement because the truck driver might not be able to identify his shortest route. It invariably will result in time waste and increased transportation cost. As the number of sub-depots and filling station has been increasing, the system getting larger and more complicated, it was impossible for a truck driver to decide the best route. Therefore, effective tools that could assist an operator to make the best decision are necessary. With an effective tool, logistics cost is expected to decrease. In order to help the project to effectively manage petroleum products purchasing system and to reduce logistics cost, this study was proposed to help solving transportation problem of transporting petroleum products from the Pipelines and Product Marketing Company (PPMC) Depot to several sub-depots and filling stations by using the MODI (modified distribution) method and the North West Corner Rule. The objective is to find optimal routes with minimum distance. (Erlander S.B 2010) By using the shortest distance, logistics or transportation cost therefore will be minimized.

Methodology

<p>| Using the mathematical model, (Ford. R.L .et. al 1956)where | M1 | M2 | M3 | M4 | CAPACITIES |
| Wᵢ denotes the source, Mⱼ the destination and Cᵢⱼ the cost of transporting from |     |    |    |    |     |</p>
<table>
<thead>
<tr>
<th>Source (W_i) and destination (M_j)</th>
<th>(W_1)</th>
<th>(W_2)</th>
<th>(W_3)</th>
<th>Destination requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUPPLY POINT TO FROM</td>
<td>20</td>
<td>100</td>
<td>70</td>
<td>40000 Ltrs</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>80</td>
<td>60</td>
<td>30000 Ltrs</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>50</td>
<td>60</td>
<td>40000 Ltrs</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>40</td>
<td>80</td>
<td>50000 Ltrs</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30000 Ltrs</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70000 Ltrs</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50000 Ltrs</td>
</tr>
</tbody>
</table>
Table 1: The Transportation Table

Using the North West rule to obtain the first feasible solution:

<table>
<thead>
<tr>
<th>SUPPLY POINT</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FROM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>W1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAPACITIES</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TESTING FOR OPTIMALITY USING THE MODIFIED DISTRIBUTION METHOD

Wi + Mj = Cij

W1 + M1 = 20
W2 + M1 = 100

TRANSPORTATION COST = (30000 * 20) + (10000 * 100) + (30000 * 80) + (30000 * 50) + (10000 * 60) + (40000 * 80) = N 9,030,000.00
\[
\begin{align*}
W2 + M2 &= 80 \\
W2 + M3 &= 50 \\
W3 + M3 &= 60 \\
W3 + M4 &= 80 \\
\text{Where } W1 &= 0 \\
W2 &= 80; W3 = 90; M1 = 20; M2 = 0; M3 = -30; M4 = -10 \\
\text{We now compute} \\
C_{ij} &= C_{ij} - (W_i + M_j) \text{ for all un-used Routes} \\
C_{12} &= 20 - (0 + 0) = 20 \\
C_{13} &= 20 - (0 + (-30)) = 50 \\
C_{14} &= 10 - (0 + (-10)) = 20 \\
C_{24} &= 40 - (40 + (-10)) = -30 \\
C_{31} &= 70 - (90 + 20) = -40 \\
C_{32} &= 60 - (90 + 0) = -30 \\
\text{Max}(C_{ij} < 0) &= -40 \\
\text{SUPLLY POINT} \\
\text{We now replace } C_{11} &= 0
\end{align*}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{TO} & \text{M1} & \text{M2} & \text{M3} & \text{M4} \\
\hline
\text{W1} & 30000 & & & 30000 \\
\text{W2} & 10000-10000 & 30000 & 30000+10000 & 70000 \\
\text{W3} & 0+10000 & 10000-10000 & 40000 & 50000 \\
\text{DESTINATION REQUIREMENTS} & 40000 & 30000 & 40000 & 40000 \\
\hline
\end{array}
\]

\[
\begin{align*}
\text{SUPLLY POINT} \\
\text{THE NEW FEASIBLE SOLUTION} \\
\text{TO} \\
\text{FROM} \\
\end{align*}
\]

\[
\begin{align*}
\text{M1} \\
\text{M2} \\
\text{M3} \\
\text{M4} \\
\end{align*}
\]

\[
\begin{align*}
\text{CAPACITIES} \\
W1 & = 20 \\
30000 \\
\end{align*}
\]
\[
\text{TRANSPORTATION COST} = (30000 \times 20) + (30000 \times 80) + (40000 \times 50) + (10000 \times 70) + (40000 \times 80) = N 8,090,000.00
\]

TEST FOR OPTIMALITY
\( W_i + M_j = C_{ij} \)
\( W_1=0; \ W_3=50; M_1=20; M_4=30. \)
Closed Routes: W2, M2, M3

We Now compute
\( C_{ij} = C_{ij} - (W_i + M_j) \)
Max\( (C_{ij} < 0) = C_{14} = -20 \)
C14 = 0

DESTINATION

<table>
<thead>
<tr>
<th>SUPPLY POINT</th>
<th>THE NEW TABLE: TO FROM</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>CAPACITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>30000-30000</td>
<td></td>
<td></td>
<td>0+30000</td>
<td>30000</td>
<td></td>
</tr>
<tr>
<td>W2</td>
<td>30000</td>
<td>40000</td>
<td></td>
<td></td>
<td>70000</td>
<td></td>
</tr>
<tr>
<td>W3</td>
<td>10000+3000</td>
<td>40000</td>
<td>30000</td>
<td>40000-30000</td>
<td>50000</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>-------</td>
<td>-------</td>
<td>-------------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>DESTINATION REQUIREMENTS</td>
<td>40000</td>
<td>30000</td>
<td>40000</td>
<td>40000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SUPPLY POINT

THE NEW FEASIBILITY SOLUTION
TO
FROM

M1
M2
M3
M4

CAPACITIES

W1

80
10
30000

30000
W2

80
30000
50
40000

70000
W3

70
40000

10000

DESTINATION REQUIREMENTS

40000
30000
40000
40000

TOTAL COST = (30000 * 10) + (30000 * 80) + (40000 * 50) + (40000 * 70) + (10000 * 80) = N 8,030,000.00

TEST FOR OPTIMALITY

COMPUTE Wi + Mj for all Used Routes

W1=0; W3=70; M3=10; M1=0
Closed Paths = W2,M2,M3
Compute Cij for all Un-used Routes
Cij = Cij – (Wi +Mj)
Optimum Solution has been reached
Since All (Cij >= 0)

SHORTEST ROUTES WITH MINIMIZED COST:
From W1 to M4 = 30000 litres
From W2 to M2 = 30000 litres
From W2 to M3 = 40000 litres
From W3 to M1 = 40000 litres
From W3 to M4 = 10000 litres
Minimized Transportation Cost = N8, 030,000.00

ALGORITHM
NORTH-WEST Corner Rule:
Step 1. Starting from the northwest corner of the transportation tableau, allocate as much quantity as possible to cell (1,1) from Origin 1 to Destination 1, within the supply constraint of source 1 and the demand constraint of destination 1.
Step 2. The first allocation will satisfy either the supply capacity of Source 1 or the destination requirement of Destination 1.
   • If the demand requirement for destination 1 is satisfied but the supply capacity for Source 1 is not exhausted, move on to cell (1,2) for next allocation.
   • If the demand requirement for destination 1 is not satisfied but the supply capacity for Source 1 is exhausted, move to cell (2,1)
   • If the demand requirement for Destination 1 is satisfied and the supply capacity for Source 1 is also exhausted, move on to cell (2,2).
Step 3. Continue the allocation in the same manner toward the southeast corner of the transportation tableau until the supply capacities of all sources are exhausted and the demands of all destinations are satisfied.
The MODI method:
Step 1. To compute the values for each row and column, set
   \[ Wi + Mj = Cij \text{ but only for those squares that are currently used or occupied.} \]
Step 2. After all equations have been written, set W1 = 0.
Step 3. Solve the system of equations for all R and K values.
Step 4. Compute the improvement index for each unused square by the formula improvement index
   \[ (Lij) = Cij - (Wi + Mj) \]
Step 5. If Lij < 0 then Max (Lij<0) = 0 goto step 1
   Else
Step 6. Optimum solution reached, display shortest route and the minimized total cost of transportation.

Conclusion
This study employed mathematical technique to solve management problems and make timely optimal decisions. (Veena 2009) If the PPMC managers are to employ the proposed mathematical model it will assist them to efficiently plan out its transportation scheduled at a minimum cost.
We recommend the solution of large-scale transportation problems using the modified distribution method and the North West Corner Rule. This proposed method is applicable to any transportation problem.
Based on the results and findings of this study, we recommend to the management of Pipeline and Products Marketing Company (PPMC) to seek to the application of mathematical theories into their operations as a necessary tool when it comes to decision making, not only in the area of logistics(the transportation Problem), but in production as well as administration.
The modified distribution method and the north-west rule are forms of optimizing the cost of transporting petroleum products from source to destination by reducing the total transportation cost and also finding the shortest route for transporting the products. This software is also designed with the objective of creating a more efficient and effective method of solving transportation problem.

The transportation cost is an important element of the total cost structure for any business. (Eykhoff, 2007). The transportation problem was formulated as a Mathematical Model and solved with the Modified Distribution and North West Rule solvers to obtain the optimal solution. The computational results provided the minimal total transportation cost and the values for the decision variables for optimality. (Orms 2010). Upon solving the transportation problems by the computer package, the optimum solutions provided the valuable information such as sensitivity analysis for Pipeline and Products Marketing Company (PPMC) : to make optimal decisions, through the use of this mathematical model (Transportation Model) , PPMC can identify easily and efficiently plan out its transportation schedules, so that it can not only minimize the cost of transporting petroleum products but also create time utility by reaching the petroleum products at the right place and right time. (Klingman D et.al 2005) This will enable them to meet the corporative objective such as education fund, and other support they offer to people of Nigeria.

REFERENCES

PROGRAM SOURCE CODE
//PROGRAM TRANSPORTATION
#include <stdio.h>
#include <math.h>
#include <iostream.h>
#define N 10

double UTC[N][N], RD[N][N], PS[N][N], SQ[N], DQ[N];
int FP[5][2*N+5], DL, ND, F, IOPTIMAL, NS, AA, BB, TPL, KM;
double PC, TC, OP, TQ, MPC;

void Init() {
    int I,J;
    cout<<"MODELLING THE TRANSPORTATION OF PETROLEUM PRODUCTS FROM
SOURCE TO DESTINATION"<<endl<<endl;
    cout<<"xxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxxxxxxx"<<endl<<endl<<endl;
    cout<<"THE TRANSPORTATION MODEL

"<<endl<<endl;
    cout<<"ENTER THE NUMBER OF SOURCE DEPOT ? "; scanf("%d", &NS);
    cout<<"ENTER THE NUMBER OF DESTINATION DEPOT ? "; scanf("%d", &ND);
    cout<<"n INPUT THE AVAILABLE SOURCE QUANTITIES READY FOR SUPPLY:n";
    for (I=1; I<=NS; I++) {
        printf(" SOURCE DEPOT #%d ? ", I); scanf("%lf", &SQ[I]);
    }
    printf("n INPUT THE REQUIRED DESTINATION QUANTITIES WAITING FOR
SUPPLY:n");
    for (I=1; I<=ND; I++) {
        printf(" DESTINATION DEPOT #%d ? ", I); scanf("%lf", &DQ[I]);
    }
    printf("n INPUT TRANSPORTATION COST PER LITRE:n");
    for (I=1; I<=NS; I++)
        for (J=1; J<=ND; J++) {
            printf(" FROM SOURCE #%d TO DESTINATION #%d ? ", I, J);
            scanf("%lf", &UTC[I][J]);
        }
    cout<<endl<<endl<<endl<<endl;
    printf("n THE OPTIMIZED SOLUTIONn");
}

void Nw();
void Optimal();
void Total();
void Search(int I, int J);
void Increase(int I, int J);
void SubMain() { //MODIFIED DISTRIBUTION ALGORITHM
    Nw();
    Optimal();
    Total();
}
void Nw() {  //N-W CORNER
  //Labels: e100, e200, e300
  int I,J;
  I=1; J=1;
  e100: if (SQ[I]<=DQ[J]) goto e200;
      RD[I][J] += DQ[J]; SQ[I] -= DQ[J];
      DQ[J]=0; J++;
      goto e300;  //else
  e200: RD[I][J] += SQ[I]; DQ[J] -= SQ[I];
       SQ[I]=0; I++;
  e300: if (I<=NS && J<=ND) goto e100;
}

void Optimal() {
  //Labels: e400, e500, e600, e150
  int I,J;
  e400: MPC=0.0;
  for (I=1; I<=NS; I++)
    for (J=1; J<=ND; J++) {
      if (RD[I][J] != 0.0) goto e500;
      Search(I,J);
      Increase(I,J);
    }
  e500: if (MPC>=0.0) {
    IOPTIMAL=1;
    goto e150;
  }
  for (I=1; I<=TPL; I++) {
    AA=FP[3][I]; BB=FP[4][I];
    if (I % 2 == 0) {
      RD[AA][BB] -= TQ;
      goto e600;
    }
    RD[AA][BB] += TQ;
  }
  e600: if (IOPTIMAL==0) goto e400;
}

void Search(int I, int J) {
  //Labels: e500, e160, e260
  int I1,I2;
  for (I1=1; I1<=NS; I1++)
    for (I2=1; I2<=ND; I2++)
      PS[I1][I2]=RD[I1][I2];
  for (I1=1; I1<=NS; I1++) PS[I1][0]=0.0;
  for (I2=1; I2<=ND; I2++) PS[0][I2]=0.0;
  PS[I][J]=1.0;
  e500: for (I2=1; I2<=ND; I2++) {
if (PS[0][I2] == 1.0) goto e160;
KM = 0;
for (I1 = 1; I1 <= NS; I1++)
    if (PS[I1][I2] != 0.0) KM++;
if (KM != 1) goto e160;
for (I1 = 1; I1 <= NS; I1++) PS[I1][I2] = 0.0;
PS[0][I2] = 1.0; F = 1;
e160:;
for (I1 = 1; I1 <= NS; I1++)
    if (PS[I1][0] == 1.0) goto e260;
KM = 0;
for (I2 = 1; I2 <= ND; I2++)
    if (PS[I1][I2] != 0.0) KM++;
if (KM != 1) goto e260;
for (I2 = 1; I2 <= ND; I2++) PS[I1][I2] = 0.0;
PS[I1][0] = 1.0; F = 1;
e260:;
if (F == 1) {
    F = 0; goto e500;
}
void Increase(int I, int J) {
//Labels: e100,e500,e130,e170,e180,e230
    int I1, I2;
    FP[1][1] = I; FP[2][1] = J; AA = I; BB = J; DL = 1; PC = 0.0; OP = 999999.0;
e100: DL++; F = 0;
    for (I1 = 1; I1 <= NS; I1++)
        if (PS[I1][BB] == 0.0 || I1 == AA) goto e500;
        FP[1][DL] = I1; FP[2][DL] = BB; AA = I1; PC -= UTC[AA][BB];
        F = 1; I1 = NS;
        if (RD[AA][BB] < OP && RD[AA][BB] > 0.0) OP = RD[AA][BB];
e500:;
    if (F == 0) goto e170;
    DL++; F = 0;
    for (I2 = 1; I2 <= ND; I2++)
        if (PS[AA][I2] == 0.0 || I2 == BB) goto e130;
        FP[1][DL] = AA; FP[2][DL] = BB; AA = I2; PC += UTC[AA][BB];
        F = 1; I2 = ND; e130:;
        if (F == 0) goto e170;
        if (AA == I || BB == J) goto e100;
goto e180;
e170: printf(" DEGENERATE SOLUTION !\n");
return;
e180: if (PC > 0.0 || PC > MPC) goto e230;
    TQ = OP; MPC = PC; DL--; TPL = DL;
    for (I1 = 1; I1 <= DL; I1++)
FP[3][I1]=FP[1][I1]; FP[4][I1]=FP[2][I1];

void Total() {
    int I,J;
    TC=0.0;
    printf("TRANSPORTS:\n");
    for (I=1; I<=NS; I++)
        for (J=1; J<=ND; J++) {
            TC += RD[I][J]*UTC[I][J];
            if (RD[I][J]==0.0) goto e400;
            printf("    FROM SOURCE #%d TO DESTINATION #%d: %8.2f Litres\n", I, J, RD[I][J]);
        }
    cout<<endl<<endl;
    cout<<"MINIMIZED TRANSPORTATION COST:"<<" "<<"N"<<" TC<<"0.0";
}

int main() {
    Init();
    SubMain();
}