



ABSTRACT

The early days of surveying work of a country basically depended on the geodetic triangulation network. The development of all triangulation activity can be done systematically by establishing triangulation network. Triangulation is the one of the most important process in surveying for establishment of control points. The aim of this study is to establish second

LEAST SQUARE ADJUSTMENT OF SECOND ORDER CONTROL STATIONS USING GLOBAL NAVIGATIONAL SATELLITE SYSTEM (GNSS) SOLUTION

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Introduction

Background to the Study

Surveying in most of the developing countries in Africa and Asia, started with advent of Europeans in these countries. The practice of the profession simply followed what's obtained in these European countries. The survey practices were geared towards economic exploration and exploitations. Basically every survey is dependent on the establishment of measured controls frameworks which are treated as being free from errors, subsequent survey works are connected to these frameworks and adjusted to it, in conformity with the principle of working from whole to parts. Control surveys provides a frame work of precisely fixed points of horizontal and vertical planes which serves as a reference points to other less precise surveys. Horizontal control is the planimetric position of a fixed point, is obtain by making angular and linear measurement from a



order control stations using least square adjustment method. The objectives designed in order to achieve the aim of the study are, to carry out Global Navigational Satellite System field observation, download the collected field data and process the collected data using Spectrum Survey Office (SSO), data processing and adjusting the observations by employing rigorous computations using Least Square adjustment method and as well carry out analysis by comparing the collected coordinates and the observed coordinates of the established control points, to present the results (Final coordinates of the control points and a plan showing the positions of these control points is presented with respect to prominent features. This research work, deals with establishment of the horizontal control point network, within the study area (Bauchi metropolis). Conventional methods/GPS techniques were used to carry out the process. The least square adjustment theory was used to adjust the network of triangles. The Geodetic calculations of this task were done with respect to Everest Ellipsoid and the observation equations method of Least Square adjustments was used to adjust the triangulation network. National Grid System of Minna was used as reference system. It is recommended that comparative study of the relative accuracies between single baseline solution and GPS network adjustment methods of observation/processing should be carried out. At the end of this research, seven stations were computed and adjusted for random errors associated with the observational procedure.

KEYWORDS: *least squares adjustment, global navigation, satellite system and spectrum survey office*

stable reference objects using surveying techniques or methods such as trilateration, triangulation and traversing depending on the accuracy required as well as the purpose of the survey, the nature of the terrain and visibility between points always influence the conduct and accuracy of the survey. Vertical controls is a fixed position which determine the



height of a points with reference to specific datum, its established by making measurement of linear and angular components in vertical directions or planes using surveying methods such as spirit leveling, trigonometric leveling etc.

Positioning simply means the determination of spatial location of objects. The general principle of GPS is that the OPS receivers collect signals from orbiting satellites and uses the principle of trilateration to compute the positions, height and time. The coordinates system associated with GPS is the earth- centered WGS 84 Cartesian reference system, as the satellites coordinates are essential for computation of user's positions; any error in these values as well as the presence of other biases will directly affect the quality of the position. In the absolute or point positioning total reliance is placed on the integrity of the coordinated points within the reference system. In the relative or differential positioning with respect to another (known) point, the known point is taking as the origin of a local coordinates system. This mode uses the relative position of two GPS receivers simultaneously tracking the satellites to achieve higher accuracy, many errors affect the absolute position of two or more GPS receivers user's to almost the same extent. These errors largely cancel out when differential positioning is carried out. There are different modes of the differential positioning procedure, but all share the characteristics that the position of OPS receiver of interest is derived relative to fixed receiver whose absolute coordinates in the satellite datum are known. According to *Kufoniyi (2013)*, surveying is the science and technology of taking measurements on, above and/or under the surface of the Earth and the representation of same on plan or map using appropriate scale. However, because of the availability of modern Geo-ICT (GIS) tools and space technology, and to emphasize the need for a service-oriented professional practice, K Ayeni (2013) while quoting NIS, 1997, defined a surveyor as “a professional person with the academic qualification and technical expertise to practice the science of measurement, to assemble, assess land and geographic related information, to use that information for the purpose of planning and implementing the efficient administration of the advancement and development of such practices”.



For surveying to operate effectively, there is need to have reference framework that will be used for orientation. Control station is a small mark set immovably into the ground, such that an instrument (e.g. a total station or GPS receiver) or optical target can be set up above it, to an accuracy of about 1 mm in the horizontal plane (Aylmer, 2004). A geodetic network is a network of triangles which are measured exactly by techniques of terrestrial surveying or by satellite geodesy. In “classical geodesy” (up to the sixties) this is done by triangulation, based on measurements of angles and of some sparse distances, the precise orientation to the geographic north is achieved through methods of geodetic observation. The principal instrument used are Theodolites and Tacheometers, which nowadays are equipped with infrared distance measuring, databases, communication systems and partly by satellite links. Nowadays, several hundred geodetic satellites are in orbit, supplemented by a large number of remote sensing satellites and navigation systems like GPS and GLONASS, which was followed by the European Galileo satellites to 2013. While these developments have made satellite based geodetic network surveying more flexible and cost effective data is terrestrial equivalent, the continued existence of fixed point networks is still needed for administrative and legal purposes or local and regional scales. Global geodetic networks cannot be defined to fixed, since geodynamics are continuously changing the position of all continents by 2 to 20cm per year. Therefore, modern global networks like ETRF or ITRF show not only coordinate of their “fixed points”, but also their annual velocities (Caspary, 1987).

Control establishment is an important concept of surveying, because every survey practiced either in large or small area requires a set of control framework to fit into, i.e. vertical and horizontal controls. The vertical controls deals with determination of the height of points, the process employed is known as leveling. In achieving the elevation of a point above a given datum, classical methods employed are Bathymetric, trigonometric, reciprocal and spirit leveling. But recently we can use total station and Global Positioning System (GPS), for height determination. Control establishment provides these requirements in several stages. The major concern is that at each stage in the hierarchy, the network can



be improperly defined within the existing network. However, since several techniques exist which lead to rigorous solutions this research will examine the techniques and the rigor of the solution in-content of practicality and economy (Dermanis 2019).

Global Positioning System (GPS), which was originally set up as a military navigation aid by the USA in the mid-1980s, but which has now become a significant tool for civilian use in general and surveyors in particular. Using differential GPS (DGPS), in which data recorded by a receiver at a 'known station' are combined with data recorded simultaneously by a second receiver at a new station which might be 30km away, it is possible to find the position of the second receiver to within about 5mm. The advantage of GPS compared to all earlier methods of surveying is that the two stations do not need to have a line of sight between them. This means that national networks of 'known stations' no longer need to be located on high hilltops or towers but can, for instance, be positioned on the verges of quiet roads (Aylmer, 2004).

Ndukwe (2001) observe, The DGPS surveying technique is a variant of the kinematic method, the aim of which is to eliminate errors in a GPS receiver to make the output more accurate. The technique is based on the principle in which the receiver's remains stationary at a reference station (point of known position) logging data continuously while the second (rover) is placed at the other station. In other word, if a GPS receiver (called base station) is placed at location for which coordinates MIX-known. The difference between the known coordinates and the GPS-calculated coordinates is the error. This error, which the base station has determined, can be applied to other GPS receiver (rover). The underlying principle is that most of the errors seen by GPS receivers in a local area will be common errors. Different methods or modes of observation exist. For a given GPS receiver, the attainable accuracy depends upon several factors such as the measurement mode, the geometric strength of solution i.e. the geometry of the satellites used, favorableness of the ionosphere, length of observation time, etc. Also, achievable accuracy depends upon the type of receiver used, for example, geodetic receivers give better accuracy than the hand-held ones. They can attain sub-centimeter accuracy, but they are very costly, the accuracy of GPS



surveying can be improved by adopting a particular mode of observation. However, for cost effectiveness, the type of receiver used and selection of observation technique should be in accordance with a project's particular requirement in terms of desired accuracy. Ndukwe (2001).

According to Monteiro et al (2005), stated that the United States Federal Radio navigation plan and IALA recommendation on the performance and monitoring of DGNS service in the band 283.5 to 325 KHz cite the United States Department of Transportation 1993 estimated error growth of 0.62 meter per 100 kilometer from the broadcast site but measurement of accuracy across the Atlantic, in Portugal suggest a degradation of just 0.22 meters per 100 kilometers. Accuracy is defined as the degree of perfection attained in the determination of a quantity. In the establishment of horizontal control networks, it is convenient and sufficient to define the minimum accuracy of a line as the ratio of the standard error of that line to the length of the line. The accuracy standard are presented in two forms, first fraction and then in part per million enclosed in bracket (PPM). The second order class 1 surveys provides the first breakdown of the basic network, and those widen the whole and released point needed to connect local urban and rural control survey whose accuracy are not lower than 1: 10000.

The second order class 2 is specifically meant to include the existing interstate cadastral framework traverses, and to provide the accuracy standard of ground control points for medium and large scale map series covering the entire country. It is therefore the appropriate order of work for control supplementation, and extension to area not covered by the second order class 1 scheme.

GPS is using constellation of satellite orbiting the earth at a radius of 26,600km on an altitudes of 20,200km, at an angle of inclination of 55° on the equator, minimum of four satellite are required to broadcast the precise travel time of the signal necessary to determine the distance, or so-called range, to the satellite, the travel time of the signal will be roughly 0.07 sec after the receiver generates the same signal. If this time delay between the two signals is multiplied by the signal velocity (speed of light in a vacuum) c , the range to the satellite can be determined from



where r is the range to the satellite and t the elapsed time for the wave to travel from the satellite to the receiver. From distance observations made to multiple satellites, receiver positions can be calculated via this relationship $r = c * t$.

For highest accuracy, for example geodetic control surveys, static surveying procedures are used. In this procedure, two (or more) receivers are employed. The process begins with one receiver (called the base receiver) being located on an existing control station, while the remaining receivers (called the roving receivers) occupy stations with unknown coordinates. For the first observing session, simultaneous observations are made from all stations to four or more satellites for a time period of an hour or more depending on the network length. (Longer networks require greater observing times.) Except for one, all the receivers can be moved upon completion of the first session. The remaining receiver now serves as the base station for the next observation session. It can be selected from any of the receivers used in the first observation session. Upon completion of the second session, the process is repeated until all stations are $\emptyset X, \emptyset Y$,

Nwilo et al, 2012 carried out GPS observations on some existing Nigerian Primary Triangulation stations, while some stations were re-established. These GPS geodetic network together with its reference frame must be continually upgraded to provide accessibility to high accuracy GPS control. Thus, a GPS campaign was carried out from October 2010 to April 2011. A total of 60 stations were observed for a period of 48 hours to form the strengthening network. These stations were even distribution throughout the GPS Network so as to connect the existing Nigerian Primary Triangulation Network to the Zero Order Geodetic Network (NIGNET) and thus defining a new Nigerian Primary Geodetic Network (NPGN) based on NGD2012 reference frame.

The observed data from the sixty (60) GPS monuments were processed using the same NIGNET stations processing procedure. The strengthening of the network involved two stages of network adjustment namely, the free network and the heavily constrain network adjustment. In the constrained adjustment, NIGNET stations held fixed



to adjust the observed network vectors to obtain the link station's coordinates to conform to NGD2012.

Quality assessment for network shows that differences less than 10 mm is achieved. Only one station in NPGN could not be processed due to poor data quality. The final heavily constrained adjustment used 11 NIGNET stations as fixed with the introduction of their respective standard deviation from the previous adjustment (Fixed NIGNET stations). This strategy allowed the GPS vectors to rotate throughout the network.

Inadequate precise control points covering the study area necessitated carrying out the establishment of second order controls and its subsequent adjustment within Bauchi metropolis that would be used for referencing of new survey works, such as monitoring structures deformation, Planning and execution of engineering works etc. Also Record has shown that of all the second order controls established within and around Bauchi metropolis none of them was established using network observation and no rigorous adjustment were done.

Aim of the Study

The aim of this research is to carry out second order control establishment in Bauchi metropolis using Network observation and Adjustment method.

Objectives of the Study

To achieve the above mentioned aim, the research has been divided into the following objectives that shall be addressed in this study

- i. To carry out GNSS field observation, download the collected field data and process the collected data using Spectrum Survey Office (SSO)
- ii. Data processing and adjusting the observations by employing rigorous computations using Least Square adjustment method and as well carry out analysis by comparing the collected coordinates and the observed coordinates of the established control points



- iii. To present the results (Final coordinates of the control points and present a plan showing the positions of these control points with respect to prominent features.

Justification of the Study

Basically every survey is dependent on the establishment of measured Frame work or controls which are treated as being free from errors, subsequently, survey works are connected to these frameworks and adjusted to it, hence there is need to establish and densify precise second order controls points that would be used for the following purposes. Evaluation of amount of error quantity in a traverse surveys, restricting error to a particular framework of the survey, adjustment and application of correction on survey measured data, construction of new engineering structures etc. The significance of this project is to provide precise control points that would serve as reference frame for the location of position of natural and artificial features and for establishment of minor control that will serve as bases for commencement and closing of minor's property survey. It can also serves as basis for setting out of engineering construction such as roads, buildings, sewer, pipelines, and also serves as bases for development planning and other research work.

Materials and Methods

This deal with all procedures involve in the execution of this project in order to obtain coordinates (X, Y, Z) of each established points using Differential Global Positioning System (GRX1). It is also the various methods adopted on the field from reconnaissance stage to the data capturing stage. A Combine methods of surveying techniques were used in executing this research work, Satellite Image was used to produce the map of the study area using Arc CIS 10.3 version. Differential Global Positioning system was used to acquire relative and absolute coordinate of the control points. Handheld GPS was also used to determine the Bearing and Distances of the control point to any nearby permanent object for the purpose of control points description (witness marks survey) and survey spectrum office was used for GPS planning and post



processing of the data acquired, however the detail procedure is as follows in Figure 1.

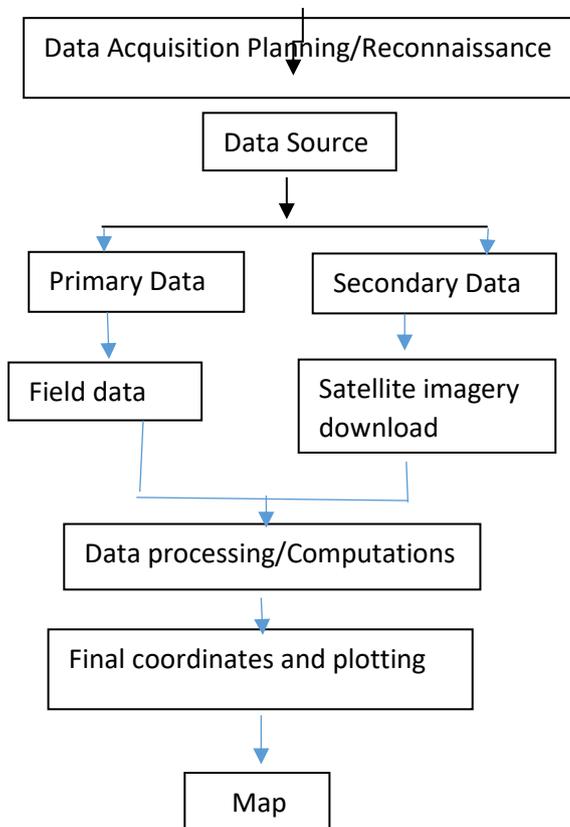


Fig 1: Workflow of Methodology

Materials used: this is divided into hardware and software

Hardware used

This consists of various hardware and software used in the course of the project. It includes the hardware and software used for data acquisition, downloading, transformation, manipulating, managing, analyzing and presentation. These hardware includes; Two (2) GPS Receivers units (SOKKIA GRX1) and their accessories, SOKKIA DGPS logger/controller, Hand held GPS and Computer system.

Software used: These software includes; ArcGIS 10.3, AutoCAD 2016, Spectrum Survey Office (SSO) and Spectrum survey Field (SSF)

Project Planning: This is the preliminary work or planning stage that must be compulsorily embarked on before the execution of the actual survey operation. This phase of the work involves gathering sufficient



information pertaining to the study so as to enable proper execution of the research which then facilitate proper decision making, it also involves having general overview of the project area in order to map out strategic way of achieving the desired goals and objectives. In this stage, various options were weighted ranging from the type of instrument to use, the reference controls, their location and extracting their coordinates from records, the materials to use, the number of days to be spent on the data acquisition, the mode in which to acquire the data that will be fast, economical and within the research specification, the day to go for each aspects or sessions of the job, the means of transportation and availability of existing map or imagery of the area of interest.

Office Planning: Office planning is the aspect that involves the compilation and study of the available information related to the project. At this stage, important decisions were taken these includes: the choice of instrument to be used as well as GPS planning for the time and method of observation was all earned out.

Field Planning: This involves visitation to the project site, and the area was critically studied. It involves determining also, location of reference controls (first and second order controls), the appropriate location for beacon emplacement. The field reconnaissance must be always preceded by working on map that will specify the shape and definition of the future control network. The project will only be fruitful if the documents used are recent and precise, it doesn't exclude the use of old documents with different scales, because their contribution is more exhaustive for the GPS project.

A pre-analysis of the visibility time of GNSS satellites was carried out in order to know the favorable lime that will enhance most visible satellite using sokia planning software such as Satellite geometry factors that has been considered when planning this project are: Number of satellites available, Minimum elevation angle for satellites (elevation mask), Obstructions limiting satellite visibility. Positional Dilution of Precision (PDOP) and Vertical Dilution of Precision (VDOP) when performing vertical OPS surveys. Static GPS surveying is perhaps the most common method of densifying project network control. Two GPS receivers are



used to measure a GPS network distance. The line between a pair of GPS receivers from which simultaneous GPS data have been collected and processed is a vector referred to as a network. The station coordinate differences are calculated in terms of a 3D, earth centered coordinate system that utilizes X-, Y-, and Z-values based on the WGS 84 geocentric ellipsoid model. These coordinate differences are then subsequently shifted to fit the local project coordinate system. GPS receiver pairs are set up over stations of either known or unknown location. Typically one of the receivers is positioned over a point whose coordinates are known (or have been carried forward as on a traverse), and the second is positioned over another point. Planning a GPS survey network scheme is similar to that for conventional triangulation or traversing. The type of survey design adopted is dependent on the GPS technique employed and the requirements of the user. A GPS network is developed to extend project control over an area. The network design establishes the stations to be occupied (new and existing) and specific networks to be observed. The network design also includes the GPS observing sequence with a given number of GPS receivers. In addition, the network design should be geometrically sound and triangles that are weak geometrically should be avoided, if possible.

Connection of Control: The control points was connected to CSB 41 at Awalah roundabout Bauchi using single line base solution and transferred to CSB 22, , for a continues observation of four hours at 15seconds epoch.

Session Planning and GPS Positioning: For the first session, the base instrument was setup at CSB 41, for the first major session, thereafter the rover was setup sequentially on each of the newly established controls, the subsequent stations on which the rover was set upon were CSB 10, CSB 17, CSB 22, CSB 44 and CSB 46, as soon as the whole loop had being observed, the base station instrument was moved to another network and the whole process was continued until the whole sessions/networks were covered. Major Session two, the base station was in CSB 22 (Tashan Babeh), while the rover was set up on CSB 41, CSB 10, CSB 17 CSB 35, and CSB 46. Major session three (3), the base station was Jos road roundabout CSB 44, while the rover stations were, CSB 41, CSB 22, CSB



35 AND CSB 46. For the minor sessions, Minor session one, the base station was CSB 17 Inkil along Gombe road, while the rover stations were CSB 10, CSB 35, CSB 22 and CSB 412, for the second minor session, the base station was CSB 35 at Dass road roundabout, while the rover stations were CSB 46, CSB 44, CSB 22 and CSB 17. Based on the planning of the research, three (3) major sessions and two minor sessions were planned and executed following the same procedures as mentioned above.

Table 1: shows the planning of the observation session and was followed to logical conclusion as follows

Table 1: Session Planning

Session No.	Base station	Rover station
Major First	CSB41 Awalah	CSB 10, CSB 17, CSB22, CSB 44, CSB 46
Major Second	CSB 22 T/Babeh	CSB 41, CSB 10, CSB 17, CSB 36, CSB 46
Major Third	CSB 44 Jos R/ABT	CSB 41, CSB 22, CSB 35 CSB 46
Minor Forth	CSB 17	CSB 10, CSB 35, CSB 22, CSB 41
Minor Fifth	CSB 35	CSB 46, CSB 44, CSB 22, CSB 17

Table 2: Project Work Schedule Stage

S/N	WORK SCHEDULE	DURATION
1	Total number of Networks	15 networks
2	Total Number of Stations (excluding reference stations)	7 Stations
3	Total Number of Reference Stations	3
4	Minimum Number of Observation on Each Stations	37mins
5	Average Networks Observable per day	5 Networks
6	Envisaged Number of days of Observation	5 days



7	Date the observation was to commence	Mon. Sept. 9, 2019
8	Date expected to Finish the Operation	Wed. Sept. 18, 2019

Before embarking on the actual acquisition of the data from the field, there was need to plan on the best way to go about the project. The study area of the project was determined from imagery, the imagery was geo-referenced and the linking roads on the study area were digitized. Each of the stations convenient for the newly established seven (7) controls was selected on the imagery then the network was designed. After the approval of the network design, all the instruments necessary for the project was determined, the number of days proposed to carry out the observation was stipulated, the total number of station in the network, the number of observation that on the average can be achievable each day on the site, the minimum observation time on each of the stations was determined. Factors that can mitigate against achieving the expected result each day, how to overcome these challenges or minimize them all these were considered in other to accomplish our desired result at the least possible time, the means of transportation specified for usage when handling the instrument, the number of personnel expected both at the reference and the rover station, and a host of other things and factors were considered before embarking on the project. Below is a table that shows the project work schedule:

Data Acquisition and GPS Observation Procedure

This deal with the determination of the X, Y, and Z coordinates of each of the established points, without the acquisition of these values, the pillars established is just an ordinary stone buried in the ground, but once observation is made on them, it automatically becomes a reference point. The GPS observation necessary for this type of control establishment was the use of survey grade differential GPS and the mode employed was static mode.



Data Processing

Arc GIS 10.3 version was used in producing control map of the project site from the acquired satellite image of the area via the following procedure includes; Image enhancement, Geo-referencing, Digitization and GPS Data Processing.

Computations/Adjustments

Finally, the network should be adjusted by Least Squares techniques not only to determine the coordinates of points but also to do a statistical analysis of the results.

Final Coordinates Extraction, Information Presentation and Reporting

Once final adjustment is done and GPS coordinates transformed into national system using an adequate approach of transformation, a final report is usually derived and may include the following, History of the project; Methodology containing the conduct of all operations field procedures, data integrity checks and the quality of the measurements and processing, final adjustment and method of transformation used; A list of points with their identification, A map of the entire network which indicate the surveyed points and the measured networks; Sheets of observations and the points description; A table presenting the final adjusted coordinates (WGS84 and local), and must be supplied in digital form; It is useful that observational data should be archived in case re-processing is required in the future.

RESULTS PRESENTATION

Following the transformation of the data, the adjustment of provisional coordinates to get the final coordinates of each individual control points established in the network was carried out. The software programme used was WOLF Pack, and it is software that was based on the theory of least square method of adjustment was used for the adjustment of data in this project. The procedure governing first and second order controls is that each of the network connecting a particular control must give the same value without any error. But no matter how hard we try it is impossible to achieve this. Hence it was necessary to carryout



adjustment computation of each of the points so as to have an error free network.

This is also known as parameters method or variation of parameters method. This equation, may be define as an equation in which each observation is expressed as a function of some unknown parameters the essential feature of a least square problem is that the number of such equation (n) be greater than the number of parameter (m) while one important feature of a least square solution is that it minimizes the sum of squares of the residual. A unique solution exists for redundant equation so far as the equation is consistent.

An adjustment model for observation equation is shown below:

L^O = Approximate value

L^A = Adjusted Observation

L^B = Original Observation

X^A = Adjusted Parameter

X^O = Approximate Value of adjusted Parameters

X = Correction to Approximate Values

V = Vector of the Residuals

A = Design Matrix

Therefore

$$X^A = X^O + X \dots \dots \dots 1$$

$$L^A = f(X^A) \dots \dots \dots 2$$

$$L^A = L^B + V \dots \dots \dots 3$$

$$L^B = Vf(X^A)$$

$$L^B + V = f(X^O + X) \dots \dots \dots 4$$

Linearizing equation (4)

$$fL^B + Vf(X^O) + F(X^O)X \dots \dots \dots 5$$

Then for a linear model,

$$f(X^O) = o$$

$$\text{Make } A = df(X^O) f(X^O)L^O \dots \dots \dots 6$$

Use equation (6) in equation (5)

$$L = L^O - L^B$$

$$L^B + V = L^O + AX \dots \dots \dots 7$$

If $X^O = o$, then $L^O = o$, and $L = -L^B$

$$\text{Therefore, } V = AX - L^B \dots \dots \dots 8$$



For a non-linear model

$$L^B + V = L^O + AX \dots \dots \dots 8a$$

$$V = AX + L^O - L^B$$

$$\text{And } L^O - L^B = L$$

$$V = AX + L \dots \dots \dots 9$$

$$\text{Let } \Phi = V^T P V$$

By direct substitution,

$$\begin{aligned} \Phi &= (AX - L^B)^T \times P (AX - L^B) \\ &= (X^T \times A^T - L^{BT}) \times (PAX - PL^B) \\ &= (X^T A^T PAX - L^{BT} PAX - X^T A^T P L^B + L^{BT} P L^B) \end{aligned}$$

$$D\Phi/DX^T = A^T PAX - A^T P L^B = 0$$

$$= A^T PAX - A^T P L^B = 0$$

$$A^T PAX = A^T P L^B \dots \dots \dots 10$$

$$D\Phi/DX^T = A^T PAX - A^T P L^B = 0 \dots \dots 10a$$

Equations (10) and (10a) is called the normal equations and $A^T P A$ is the normal coefficient which is $\neq 0$ for a unique solution

$$X = -(A^T P A)^{-1} A^T P L^B \dots \dots \dots 11$$

$$X = -(A^T P A)^{-1} A^T P L \dots \dots \dots 11a$$

To determine the standard error of the Adjusted Parameters, there was a need to know the Variance/Covariance matrix of X

$$\text{A-priori Variance} = \delta_0^2$$

$$\text{A-posteriori Variance} = \delta_0^2$$

$$\delta_0^2 = V^T P V / n - m \dots \dots \dots 12$$

Where; n = number of equation, m = number of parameter

Then Variance /Covariance matrix of adjusted observation

$$\sum X^A = \delta_0^2 (A^T P A)^{-1} \dots \dots \dots 13$$

Variance /Covariance matrix of adjusted observation

$$\begin{aligned} \sum L^A &= A \sum X^A \\ &= A \delta_0^2 (A^T P A)^{-1} \times A^T \dots \dots \dots 14 \end{aligned}$$

Observation equations are equations in which the adjusted observations are expressed explicitly as functions of some adjusted parameters. These observations are usually expressed in terms of the coordinates (parameters) of the points that form them in order to form the observation equations.



The results obtained in this project are presented in tables and figures below, the first session of observation procedure was the connection from CSB 41 at Awalah roundabout to at CSB 10 along Maiduguri road with at least four hours continues observation at 15seconds epoch, the most provable value of the co-ordinate was obtain using single network solution.

Table 3: shows the adjustment summary used for connections, GPS observation difference with precision, and adjusted control point and Figure 2 shows the COGO plotted view of the connection session using single baseline solution, however Table 4 shows the adjustment summary of Seven GPS controls network points of 16 vectors at 95% confidence level using advance t-test statistical test.

Table 3: Adjustment Summary of Control Points Used for Connection.

SERIAL NO.	BEACON ID	EASTING	NORTHING	CONTROLS USED FOR PROCESSING/ADJUSTMENT
1	CSB44	590047.579	1140989.009	NOTI 2959679.1
				DAKR 2973208.2
				RABT 3094151.9
2	CSB35	587953.212	1137543.314	MBAR 2594794.1
				YKRD 1693366.8
				NKLG 1097531.4
3	CSB22	592177.439	1140665.261	YKRD 1698104.1
				NOTI 2959666.5
				NKLG 110068.0
4	CSB17	596787.832	1139882.887	NOTI 2959713.3
				MBAR 2588034.0
				NKLG 109993.0
5	CSB41	592120.145	1143783.891	DL5972 ISKU IRAQ SURVEY KUT CORS ARP 4339260.0
				DK7831 ISNA IRAQ SURVEY NAJAF CORS ARP 4200708.9
				DP7492 ZAXO UNI ZAKHO CORS ARP 4340883.3
6	CSB10	598331.893	1145161.396	DK4117 YQXI GANDER WAAS CORS ARP 6901036.8
				DK7831 ISNA IRAQ SURVEY NAJAF CORS ARP 4195345.5
				DK6558 YYRI GOOSE BAY WAAS CORS ARP 7310265.5
7	CSB46	587385.649	1140028.055	DK6558 YYRI GOOSE BAY WAAS CORS ARP 7308246.3
				DK7831 ISNA IRAQ SURVEY NAJAF CORS ARP 4206422.7
				DK4117 YQXI GANDER WAAS CORS ARP 6898302.4

Table 4: Final coordinates of controls established

S/No	X	Y	Z	ID
1	590047.579	1140989.009	639.761	CSB44
2	587953.212	1137543.314	624.835	CSB35



3	592177.439	1140665.261	648.499	CSB22
4	596787.832	1139882.887	624.467	CSB17
5	592120.145	1143783.891	653.238	CSB41
6	598331.893	1145161.396	597.851	CSB10
7	587385.649	1140028.055	652.703	CSB46

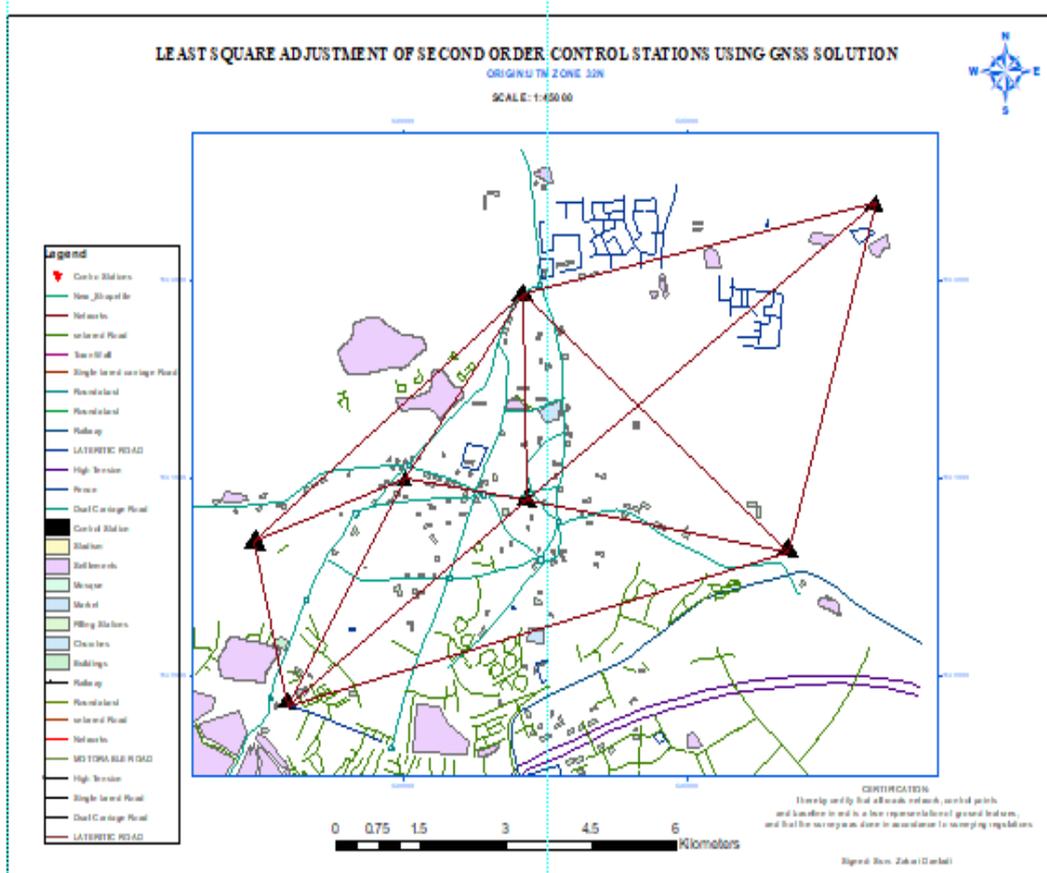


Fig. 2: Map showing Established control points

Summary

The project was successfully carried out, observation was done using differential GPS receivers. A precise static post processed network adjustment method of observation was used and sokkia survey spectrum office software was use to process the data using single baseline solution for the connection from CSB41, to CSB10 and network adjustment solution adjust the system of loops of the networks observations, prior to observation GPS planning was carried out in other to acquire data at most favorable time of good satellite geometry between receivers and the constellations in the GNSS which mean an increase in number



satellite will also increase the accuracy of GPS positioning because will decrease DOP. Witness mark survey was earned out for proper station description of the control points, the parameters acquired are the bearing and distances from the controls points to any prominent and permanent object within vicinity of the control points which will in the feature enable re-establishment of the missing control if any missing. Lastly a control map showing the location and geometric figure of the control point's network was created and an attribute table was also created and populated with the station description parameter.

Conclusion

Generally the aim of the project was successfully achieved through execution of the set objectives; control point map in the study area was produced.

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