



QUANTIFYING THE EFFECT OF INTEREST RATE ON THE SENSITIVITY VALUES OF TWO INTERACTING POPULATIONS OF INVESTORS IN A STOCK MARKET

NAFO, N. M

*Department of Mathematics, Rivers State University,
Nkpolu- Oroworukwo, Port Harcourt, Rivers State, Nigeria*

ABSTRACT

In this paper, a computational approach is used to study the effect of interest rate on the sensitivity of the growth rate of two interacting populations of investors in a stock market over a trading period in days. This work clearly shows that the growth rate of the first population of investors is a highly sensitive parameter. The details of the numerical simulation analysis are presented and

INTRODUCTION

The work of quantifying the influence of interest rate on the populations of interacting stock market investors is a daunting task in numerical simulation. We have therefore, resorted to using a technique of sensitivity analysis to estimate the impacts of the interest rate and the duration of trading on the sensitivity of stock market parameters.

Materials and Method

Our estimated stock market parameters (Nafo and Ekaka-a (2011), Ekaka-a and Nafo (2011), were based on the Abia State Stock Exchange (Okoroafor and Osu (2009). The process of varying a model parameter one-at-a time and observing its impact on the model output constitutes the notion of sensitivity analysis for a Lotka-Volterra continuous system of coupled-first order nonlinear ordinary differential equations (Crick et al (1987), Ekaka-a (2009), Ekaka-a and Nafo (2012), Ekaka-a and Nafo (2012), given as follows:

$$\frac{dw_1(t)}{dt} = w_1(a_1 - b_1 w_1^2 - c_1 w_2) \quad (1)$$



discussed quantitatively.

Keywords: Sensitivity, growth rate, stock market, interest rate, trading period, simulation analysis

$$\frac{dw_2(t)}{dt} = w_2(a_2 - b_2w_2^2 - c_3w_1) \quad (2)$$
$$w_1(0) > 0, w_2(0) > 0$$

Where;

$w_1(t)$ and $w_2(t)$ represent the wealth of the first and second populations of investors respectively at trading time t in days.

a_1 is the intrinsic growth rate of the first population of investors

a_2 is the intrinsic growth rate of the second population of investors

b_1 is the intra-specific coefficient which is the inhibiting factor due to the interaction of the first investor with itself.

b_2 is the intra-specific coefficient which is the inhibiting factor due to the interaction of the second population with itself.

c_1 is the inter-specific coefficient which is the inhibiting factor on the growth of the first investor due to its interaction with the second

c_2 is the inter-specific coefficient which is the inhibiting factor on the growth of the second investor due to its interaction with the first.

$w_1(0)$ and $w_2(0)$ are the initial wealth of the first and second investors respectively.

The solution of our proposed problem can be done quantitatively, using the method of mathematical norms (Ekaka-a (2009), Nafo and Ekaka-a (2012)). From our recent contribution on the stock market dynamics, we know that the impact of the interest rate will be clearly quantified if the intrinsic growth rates of the two interacting populations of investors are modified a little one-at-a-time over a shorter and a longer duration of trading period which are both characteristics of stock market behaviour. The key question which is usually neglected is: 'When these changes in a model parameter are made, what is the impact of these



changes on the model output (Crick et al (1987), Ekaka-a (2009))? That is, do these changes in a model parameter produce a bigger or smallest effect on the model output? The extent of the variation of a model parameter on the model output is the main trust of this present study.

Results

Our contributions and observations are displayed in Table 1. Similar results are displayed in Table 2, Table 3, Table 4 and Table 5.

Table 1: Quantifying the effect of changing interest rate on the sensitivity of stock market model when growth rate of the first population of investors denoted by the parameter 'a₁' is varied by 1% while the growth rate of the second population of investors is fixed (including unchanging values of the other model parameters)

Duration of Trading Period in Days	1-norm	2-norm	Infinity-norm	Observation
20	25.30	22.90	24.75	High Sensitivity
40	43.13	35.40	32.82	High sensitivity
60	57.87	45.32	38.52	High sensitivity
80	70.46	53.58	42.92	High sensitivity
100	81.26	60.66	46.24	High sensitivity
200	115.94	83.61	52.88	High sensitivity
300	132.80	95.01	54.06	High sensitivity
400	142.40	101.60	54.53	High sensitivity

Table 2: Quantifying the effect of changing interest rate on the sensitivity of stock market model when growth rate of the first population of investors called parameter 'a₁' is varied by 2% when the growth rate of the second population of investors is fixed (including unchanging values of the other model parameters)

Duration of Trading Period in Days	1-norm	2-norm	Infinity-norm	Observation
20	25.07	22.71	24.55	High Sensitivity
40	42.80	35.24	32.60	High sensitivity
60	57.44	45.02	38.31	High sensitivity
80	70.02	53.27	42.73	High sensitivity
100	80.81	60.35	46.10	High sensitivity
200	115.58	83.40	52.85	High sensitivity



300	132.54	94.86	54.06	High sensitivity
400	142.20	101.50	54.53	High sensitivity

Table 3: Quantifying the effect of changing interest rate on the sensitivity of stock market model when growth rate of the first population of investors called parameter 'a₁' is varied by 3% when the growth rate of the second population of investors is fixed (including unchanging values of the other model parameters)

Duration of Trading Period in Days	1-norm	2-norm	Infinity-norm	Observation
20	24.85	22.51	24.35	High Sensitivity
40	42.46	34.97	32.38	High sensitivity
60	57.06	44.72	38.10	High sensitivity
80	69.68	52.96	42.55	High sensitivity
100	80.36	60.03	45.94	High sensitivity
200	115.21	83.16	52.82	High sensitivity
300	132.27	94.70	54.06	High sensitivity
400	142.00	101.37	54.53	High sensitivity

Table 4: Quantifying the effect of changing interest rate on the sensitivity of stock market model when growth rate of the first population of investors called parameter 'a₁' is varied by 4% when the growth rate of the second population of investors is fixed (including unchanging values of the other model parameters)

Duration of Trading Period in Days	1-norm	2-norm	Infinity-norm	Observation
20	24.63	22.32	24.15	High Sensitivity
40	42.12	34.70	32.15	High sensitivity
60	56.65	44.42	37.88	High sensitivity
80	69.13	52.64	42.36	High sensitivity
100	79.90	59.72	45.78	High sensitivity
200	114.84	82.93	52.80	High sensitivity



300	132.00	94.54	54.05	High sensitivity
400	141.77	101.24	54.52	High sensitivity

Table 5: Quantifying the effect of changing interest rate on the sensitivity of stock market model when growth rate of the first population of investors called parameter 'a₁' is varied by 5% when the growth rate of the second population of investors is fixed (including unchanging values of the other model parameters)

Duration of Trading Period in Days	1-norm	2-norm	Infinity-norm	Observation
20	24.40	22.12	23.95	High Sensitivity
40	41.78	34.43	31.93	High sensitivity
60	56.23	44.11	37.66	High sensitivity
80	68.68	52.32	42.16	High sensitivity
100	79.43	59.40	45.62	High sensitivity
200	114.46	82.70	52.76	High sensitivity
300	131.72	94.37	54.05	High sensitivity
400	141.56	101.12	54.52	High sensitivity

The results in our tables above come from the consideration of an example which relates directly with stock market interacting systems (Ekaka-a and Nafo, 2011). The example concerns the dynamics of two interacting stock market populations based on the Abia State stock exchange time series data of Okoroafor and Osu, (2009)

Summary and Conclusion

Without loss of generality, we have successfully used the method of the mathematical norms [1-norm, 2-norm, infinity-norm] to quantitatively measure the impact of the interest rate on the sensitivity values of a single model parameter of a stock market model. When the growth rate parameter of the first population of investors is varied a little one-at-a-time while other parameters are fixed, we found that higher sensitivity is more associated with a longer period of trading than with a shorter period of trading. Another significant observation is that the sensitivity of the growth rate is high irrespective of the period of duration. Therefore, the growth rate parameter which is conjectured as implying the response of the interest rate should be seen as more sensitive in terms of the period of trading.



Subject to a few expected risks and penalties of stock market trading, we would jointly suggest that our present contributions are capable of complementing our previous contribution and can provide further useful insights in the tactical planning which investors can tap into. Examining the influence of changing the growth rate parameter of the second population of investors on the model output when other model parameters are fixed will be our next level of simulation on sensitivity analysis.

REFERENCES

- Crick, M.J., Hill, M.D. and Charles, D. (1987). The Role of Sensitivity Analysis in Assessing Uncertainty. In: Proceedings of an NEA Workshop on Uncertainty Analysis for Performance Assessments of Radioactive Waste Disposal Systems, Paris, OECD, Pp. 1-258.
- Ekaka-a, and Nafo, N.M. (2012). Parameter Ranking of Stock Market Dynamics: A Comparative Study of the Mathematical Models of Competition and Mutualistic Interactions, *Scientia Africana*, Vol. 11, (1), Pp. 36-43
- Ekaka-a, E.N, (2009). Computational and Mathematical Modeling of Plant Species Interactions in a Harsh Climate, Ph.D. Thesis, Department of Mathematics, The University of Liverpool and the University of Chester, United Kingdom.
- Ekaka-a, E.N, and Nafo, N.M. (2012). Stabilizing a Mathematical Model of Stock Market Population System, *Scientia Africana*, Vol. 11, (1), Pp. 92-97
- Ekaka-a, E.N. and Nafo, N.M. (2011). Numerical Parameter Estimation: Application to Stock Market Dynamics, NMC Conference on Scientific Computing, Abuja, Conference Proceedings, No. 34, Pp. 21
- Nafo, N.M. and Ekaka-a, E.N. (2011). Modelling Mutualism in a Stock Market Dynamics: Application within the Niger Delta Financial Markets, NMS Minna Conference Proceedings, Minna, No. B49.
- Okoroafor, A.C. and Osu, B.O. (2009). An Empirical Optimal Portfolio Selection Model, *African Journal of Mathematics and Computer Science Research*, 2(1), 001-005.