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Using Mathematical Model To Assess The Impact Of Vaccination Of Measles

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Abstract

This paper presents mathematical model to investigate the declination of measles due to vaccination. The primary aim of this study is to increase global understanding and ability to prevent death and illness due to measles. We employed an SEVIR model which includes; susceptible, exposed, vaccinated, infected and recovered compartments. Stability analysis for Disease Free Equilibrium and Epidemic Equilibrium have been carried out to evaluate the impact of the vaccination method of control. We finally recommended that Immunization organisations in collaboration with government ought to reinforce routine vaccination system and effort should be intensified towards decreasing the level of contact rate.

Introduction

The most useful experimental tool for building and testing theories in the contemporary world is Mathematical modelling. It plays vital role towards assessing quantitative conjectures, answering specific questions, determining sensitivities to changes in parameter values, and estimating key parameters from data. Models are used for understanding the transmission characteristics of infectious diseases in communities, regions, and countries which can lead to better approaches to decreasing the transmission of these diseases (Hethcote, 2000). Measles is a profoundly infectious and genuine sickness brought about by an infection. Prior to the presentation of antibody for measles in 1963 and far reaching immunization, significant pandemics happened around each 2–3 years and measles caused an expected 2.6 million people passes away every year. In excess of 140,000 individuals kicked the bucket from measles in 2018. These are mostly children younger than 5 years, regardless of the safe and effective vaccine (WHO, 2019).

The classical signs and symptoms of measles include fever, cough, headache, cold, red eyes, and rashes. The rashes begin several days after the fever starts. It produces also a characteristic red rash and can lead to serious and fatal complications including pneumonia, diarrhea, and encephalitis (Ejima *et al*, 2012). Stone L, Shulgin B. and Agur Z. (2000) propose a pulse vaccination strategy of measles using an SIR epidemic model where it was observed that vaccinating the susceptible population repeatedly in a series of pulse; the measles infection may be eradicated from the entire model population.

Those present circumstances concerning the prospects about measles destruction over Israel invigorated the handling for new rules to measles immunization. The first dosage may be prescribed during 15 months of age. Furthermore, a second measurement at long period of 6 years. These rules are in light of those traditional ideas of a consistent immunization exertion every year. For such strategies, immunization influences the plentifulness and the time of the epidemics. However, it doesn't motivate an extremely serious difficulty of the common Progress for transmission.

Clinched alongside contrast, the hypothesis of population dynamics over barbarously fluctuating situations infers that at the ecological example forced on the populace takes those manifestation of discrete scenes of devastation, it is the dividing from claiming these scenes that determines number hold on it.

In view of this hypothesis we inspected the theory that measles epidemics could make a greater amount effectively regulated when the natural temporal process of the epidemics is antagonized by another temporal process, by an immunization effort that varies significantly and abruptly in time. This arrangement is referred to as pulse immunization and develops mathematical models suggesting that pulse immunization about kids age-old 1-7 years, once every 5years, might sufficiency for keeping repetitive epidemics to Israel (Agur *et al*. 1993).

David *et al.* (2000) Apart from their public health importance, epidemics of youth infections have Gave profitable insights under hypotheses from claiming number flow. When impostor immunization started in the 1960s, epidemics from claiming measles exhibited both general and unpredictable flow. To nations the place impostor inoculation projects need aid Right away done place, measles epidemics need ended up additional unpredictable concerning illustration general frequency need declined. In the United Kingdom, mass immunization has also coincided with a sharp reduction in the geographical coherence of measles epidemics.

A number of authors have put up some studies using mathematical modeling about the transmission dynamics of measles (Simons *et al*, 2012, Mossong & Muller, 2003, Trottier & Philippe, 2003, Allen *et al*, 1990, Allen *et al*, 1991, Babad *et al*, 1995, Bauch & Earn 2003, Wolfson *et al*, 2007). Some of these models are developed to explain the size and duration of measles epidemic in a given population. The purpose of the current study is to extend some of the abovementioned studies, by designing and analyzing a new comprehensive model for measles transmission in a population that incorporates the role of vaccine and treatment. The purpose of the study is to qualitatively assess the impact of vaccination and treatment of measles in a population.

Statement of the problem

Eradication of measles has been sustained in the United States since eradication was chosen nearly 15 years ago. However, approximately 20 million instances of measles happen every year Globally, and importation into the United states keep on posing a danger for measles cases and flare-up among unvaccinated persons (Robert *et al*. 2015).

In spite of the fact that the antibodies are said to be 99% compelling, a late hostile to immunization development in the United States has erroneously connected a mental imbalance to the immunizations. Researchers have altogether censured the cases as false and misleading. In any case, many have connected the development to the record number of measles cases in 2014. A year ago, 644 cases were affirmed, representing an almost two-decade high.

Objective of the study

The key objectives of this study are as follow:

- To develop a mathematical model for the control of measles given emphasis to vaccines.
- To increase global understanding and capacity to prevent death and illness from Measles.

Methodology

This research work is a quantitative study that involves deterministic approach of differential equations. Stability analysis has been carried out using Jacobian Matrix to assess the stability of the model.

Model formulation

As the first step in modelling process, we identify the independent and dependent variables. The independent variables is time measured in days, months or years depending on the disease under study and the variables count people in each of the groups, each as a function of time where;

S =S (t), Number of susceptible individual at time t

$E = E(t)$, Number of exposed individual at time t

$V = V(t)$, Number of immunised individual at time t

$I = I(t)$, Number of infected individual at time t

$R = R(t)$, Number of recovered individual at time t

$N = N(t)$, Total population at time t

Thus,

$$S(t) + E(t) + V(t) + I(t) + R(t) = N(t)$$

Assumptions

The following assumptions are used in order to build the model

(a.) The number of infected people increases proportional to both the number of infectious and the number of susceptible i.e. $\frac{\beta SI}{N}$ with $\beta > 0$. So that number of susceptible decrease at the same rate. Here, β is called the effective infectious rate.

(b.) The rate of removal of infectious to recovered compartment is proportional to the number of infectious only i.e. ΥI with $\Upsilon > 0$. This is called the removal rate.

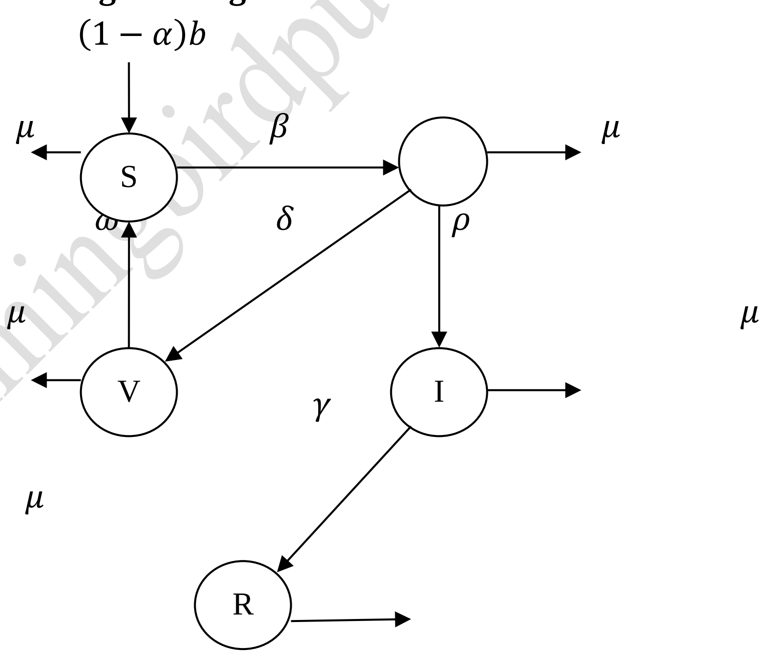
(c.) A person can die at any of the compartment. Therefore, μ is taken as natural death.

(d.) Every individual born fall into the susceptible compartment. Only those that are successfully vaccinated are removed and immune.
(e.) The population is entirely Homogenous
(f.) We also assume that there is constant birth and death rate.

Parameters

The parameters used are as follows:
β : −Contact rate
$\frac{1}{\gamma}$: −Iinfectious period
b : −Birth rate
ω : −The rate at which vaccinated individuals move back to susceptible population
μ : − Mortality rate
α : − Represents the proportion of new birth given vaccines to protect against infection
$(1-\alpha)b$: −Rrepresents the population of individuals not immunized against infection
αb : − Represents the proportion of incoming individuals immunized against infection
R_0 : − Basic reproductive number

Epidemiological Diagram for the model



Taking into account the above assumptions with the parameters, this vaccination model is governed by the differential equations as follows:

$$\frac{dS}{dt} = (1-a)b + \omega V - \beta SI - \mu S$$
$$\frac{dE}{dt} = \beta SI - \rho E - \delta E - \mu E$$
$$\frac{dV}{dt} = \delta E - \omega V - \mu V \qquad 1.0$$
$$\frac{dI}{dt} = \rho E - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

At steady states,

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dV}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \qquad 1.1$$

This gives:

$$(1-a)b + \omega V - \beta SI - \mu S = 0 \qquad 1.2$$
$$\beta SI - \rho E - \delta E - \mu E = 0 \qquad 1.3$$
$$\delta E - \omega V - \mu V = 0 \qquad 1.4$$
$$\rho E - \gamma I - \mu I = 0 \qquad 1.5$$
$$\gamma I - \mu R = 0 \qquad 1.6$$

From (1.2):

$$(1-a)b + \omega V = \beta SI + \mu S$$
$$(1-a)b + \omega V = S(\beta I + \mu)$$
$$S = \frac{(1-a)b + \omega V}{\beta I + \mu} \qquad 1.7$$

Zero steady states are

For $I = 0, V = 0,$

$$S = \frac{(1-a)b}{\mu} \qquad 1.8$$

From (1.3): for $I = 0,\ E = 0$

From (1.4): for $E = 0,\ V = 0$

From (1.5): for $E = 0, \ I = 0$
From (1.6): for $I = 0 \ and \ V = 0$ then $R = 0$
Hence the DFE is when

$$S^{\star} = \frac{(1-a)b}{\mu}$$
$$E^{\star} = 0$$
$$V^{\star} = 0$$
$$I^{\star} = 0$$
$$R^{\star} = 0$$

Now, the non-zero steady states are:

From (1.3):

$$E = \frac{\beta SI}{\rho + \delta + \mu} \qquad 1.9$$

From (1.4):

$$E = \frac{(\omega + \mu)V}{\delta} \qquad 2.0$$

From (1.5):

$$E = \frac{(\gamma + \mu)I}{\rho} \qquad 2.1$$

Equating (1.9) and (2.1) we get

$$S = \frac{(\rho + \delta + \mu)(\gamma + \mu)}{\beta\rho} \qquad 2.2$$

Equating (2.0) and (2.1) gives

$$I = \frac{\rho(\omega + \mu)V}{\delta(\gamma + \mu)} \qquad 2.3$$

Substituting equation (2.2) and (2.3) in equation (1.2) we get

$$V = \frac{\delta\mu(\rho + \delta + \mu)(\gamma + \mu) - (1-\alpha)b\beta\rho}{\beta\rho\{\omega\rho - (\rho + \delta + \mu)(\omega + \mu)\}} \qquad 2.5$$

Substituting equation (2.5) in equation (2.0) gives

$$E = \frac{(\omega+\mu)\{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho\}}{\beta\rho\delta\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}} \qquad 2.6$$

Substituting equation (2.5) in equation (2.3) gives

$$I = \frac{(\omega+\mu)\{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho\}}{\beta\delta(\gamma+\mu)\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}} \qquad 2.7$$

From (1.6):

$$R = \frac{\gamma I}{\mu} \qquad 2.8$$

Substituting equation (2.7) in equation (2.8) gives

$$R = \frac{\gamma(\omega+\mu)\{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho\}}{\beta\delta\mu(\gamma+\mu)\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}} \qquad 2.9$$

Hence the Epidemic Equilibrium is when

$$S^* = \frac{(\rho+\delta+\mu)(\gamma+\mu)}{\beta\rho}$$

$$E^* = \frac{(\omega+\mu)\{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho\}}{\beta\rho\delta\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}}$$

$$V^* = \frac{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho}{\beta\rho\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}}$$

$$I^* = \frac{\rho(1-\alpha)b}{(\gamma_1+\mu)(\rho+\delta+\mu)}$$

$$R^* = \frac{\gamma(\omega+\mu)\{\delta\mu(\rho+\delta+\mu)(\gamma+\mu)-(1-\alpha)b\beta\rho\}}{\beta\delta\mu(\gamma+\mu)\{\omega\rho-(\rho+\delta+\mu)(\omega+\mu)\}}$$

Basic Reproduction Number

The basic reproduction number denoted byR_0, refers to the number of new cases of infection linked to a person infected shortly after the pathogen was introduced into population with no pre-existing immunity. Generally, the higher the value of R_0, the harder it is to control the epidemic. When$R_0 < 1$, the infection will die out in the long run. But when$R_0 > 1$, the infection will invade.

Form the model, since the term γ refers to the removal rate, $b\ and\ \mu$ are the birth and death rates respectively, then the average infectious period is $\frac{1}{\gamma+\mu+b}$. Since the rate of transmission per infective is β then the basic reproduction number, R_0 will be $R_0 = \frac{1}{\gamma+\mu+b} \times \beta$

By the Threshold,

$$\frac{1}{R_0} = \frac{\gamma + \mu + b}{\beta} \qquad (3.0)$$

This implies,

$$R_0 = \frac{\beta}{\gamma + \mu + b} \qquad (3.1)$$

Numerical Calculation of R_0

Table 1.1 shows the two values of R_0 based upon the model system

S/N	β	γ	μ	b	R_0
1.	0.1455	0.3234	0.0005	0.0341	0.4064
2.	0.0742	0.0315	0.0005	0.00542	1.9829

From table 1.1, for these parameter values, the basic reproduction number for the Disease-Free Equilibrium (DFE) is $R_0 = 0.4064 < 1$.This shows that the infection is temporal and the disease dies out in time. If we keep the value of μ unchanged and change the values of $b, \beta\ and\ \gamma$then the basic reproduction number is calculated as $R_0 = 1.9829 > 1$and the disease becomes endemic. However, an average infectious individual is able to replace itself and the number of infected rises and an epidemic reveal.

Stability Analysis

In epidemiology if a system is stable under any condition(s), then the disease can be eradicated under such condition in future time. the Jacobean associated with (1.0) at the equilibrium points (SEVIR) is

$$J(S^*, E^*, V^* I^*, R^*)$$

That is,

$$J = \begin{pmatrix} -(\beta I + \mu) & 0 & \omega & -\beta S & 0 \\ \beta I & -(\rho + \delta + \mu) & 0 & \beta S & 0 \\ 0 & \delta & -(\omega + \mu) & 0 & 0 \\ 0 & \rho & 0 & -(\gamma + \mu) & 0 \\ 0 & 0 & 0 & \gamma & -\mu \end{pmatrix}$$

The characteristics equation of the Jacobian matrix is

$$|J - \lambda I| = \begin{vmatrix} -(\beta I + \mu) & 0 & \omega & -\beta S & 0 \\ \beta I & -(\rho + \delta + \mu) & 0 & \beta S & 0 \\ 0 & \delta & -(\omega + \mu) & 0 & 0 \\ 0 & \rho & 0 & -(\gamma + \mu) & 0 \\ 0 & 0 & 0 & \gamma & -\mu \end{vmatrix} = 0$$

$$(\beta I + \mu + \lambda)\{-(\rho + \delta + \mu + \lambda)[-(\omega + \mu + \lambda)\{-(\gamma + \mu + \lambda)(-\mu - \lambda)\}]\} = 0$$

⇒ At the DFE state,

$$\beta I + \mu + \lambda = 0$$
$$\Rightarrow \lambda = -\beta I - \mu$$

Putting $I = 0$,

$$\lambda = -\mu$$
$$\therefore \lambda_1 = -\mu$$

Also,

$$-(\rho + \delta + \mu + \lambda) = 0$$
$$\Rightarrow \lambda = -\rho - \delta - \mu$$
$$\therefore \lambda_2 = -\rho - \delta - \mu$$

Then,

$$-(\omega + \mu + \lambda) = 0$$
$$\Rightarrow \lambda = -\omega - \mu$$
$$\therefore \lambda_3 = -\omega - \mu$$

Also,

$$-(\gamma + \mu + \lambda) = 0$$
$$\Rightarrow \lambda = -\gamma - \mu$$
$$\therefore \lambda_4 = -\gamma - \mu$$

And

$$-\mu - \lambda = 0$$
$$\Rightarrow \lambda = -\mu$$
$$\therefore \lambda_5 = -\mu$$

Since $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4\ and\ \lambda_5$ are all negative, it shows that the disease will completely die out in time.

Conclusion

In this study stability analysis of the model of vaccination for Measles has been investigated to understand its impact as a method of control using an SEVIR Mathematical model. It was successfully proved that all the five eigenvalues are negative which indicates that the disease-free equilibrium state is stable. At this point, it is relevant to say that measles will be completely eradicated in time. Therefore, it was finally recommended that Immunization organisations in collaboration with government ought to reinforce routine vaccination system and effort should be intensified towards decreasing the level of contact rate.

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