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Application of Queuing Model to the Customer Management in a Banking System

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Keyword:
Average number of customers in the queue (LQ), Average number of customers in the system (LS), I.C.T, First bank. System, Waiting time.

ABSTRACT
The application of queuing model is of immerse important in the banking system which include reducing waiting time (queue) which happen to be the most challenge in the banking system. In view of the wide spread economic climate in the country, there is a greater used to manage our time as well as to minimize the lost of customers, it's also significant to highlight the use of queue model in view of competition. This research consider one week of two working days from 8:15 – 10:45 of a total time of 2hours 18minutes at the withdrawal section of First Bank Bauchi main branch, in which M/M/4 was considered where the arrival time of the customer is exponentially distributed and the service time also follows an exponential distribution in which four servers were considered, a first come first serve basis was also observed. The capacity of the system is infinite, customer arrival rate and service rate was observed, the probability that servers are idle and busy, the average number of customers in the queue (LQ) and also in the system (LS) was obtained, the research observed the problem of queue in the banking system and it was discovered that most reasons that make customers to queue in the bank is as a result of lack of enough servers and Information Communication Technology (I.C.T) application. If servers and

Information Communication Technology (I.C.T) application is in a good condition, customers will be serve as they arrive.

Introduction:

Population of customers can be considered either limited (closed system) or unlimited (open system) unlimited population represent a theoretical model of systems with a large number of possible customers (in bank on a bus. street, a motorway petrol station) example of a limited population may be a number of processes to be run (served) by a computer or a certain number of machine to be repaired by a service man, it is necessary to take the term" customer" very generally, customer may be people, machine of various nature, computer process, telephone cells e.t.c (Bhat 2008)

A queue represents a certain number of customers waiting for service (of course the queue. Maybe empty). Typically the customer being served is considered not to be in the queue. Sometime the customers from a queue literally (people waiting in a line for a bank teller). Sometime the queue is an abstraction (planes waiting for a run way to land). These are two important properties of queue maximum size and queue discipline) (Bhat 2008)

It now becomes one of the challenges for banks to be able to manage the time spent by customers in the banking hall to remain competitive. The aim of this research is to minimize waiting time in queue by proper queue management and they by maximizing through put. We develop a web based application that assign each customer queue number on arrival based on touching the screen and the queue number are stored electronically. (Lekara and Odiri 2014)

First in, first out queue method is implemented in the design to achieve an orderly service delivery, also customer who have successful gotten the queue number are attended to first based on FIFO queue model already programmed. After a successfully daily operation in the bank performance measure can be display.

The process system when implemented will minimize the problems of congestion, and better service will be achieved. This research uncovered the applicability and extent of usage of queuing models in achieving customer satisfaction at the lowest cost.

Queuing theory is mathematical study of waiting in the line [queue]. The theory enable mathematical analysis of several related processes, including the arrival at the [bank of the] queue, waiting in the queue [essentially a storage process] and being served at the front of the queue.

The queue permits the derivation and calculation of several performance measure including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states such as empty, full, having an available server or having to waiting a certain time to be served(Bronson 2001)

Queuing theory has become one of the most important available and arguable one of the most universally used tools by an operation researcher. It has application in diverse fields including telecommunication, traffic, and engineering, computing and designing of factories, shops, offices, banks and hospital. [Bronson 2001]

A queuing model of a system is an abstract representation whose purpose is to isolate those factors that relate to the system's ability to meet service demand whose occurrence and durations are random (SZtrik J. 2010)

Queuing theory deals with problems which involves queuing (or writing) typical example might be

i. Banks (supermarkets-waiting for service)
ii. To Computers- waiting for response, failure situation in waiting for failure to occur e.g. in a piece of machinery (Jackson 1989).
iii. Public transport- waiting for train or a business

As well as known queues are common every day experience, queues form. Because resources are limited. Infact at makes economic sense to have queues. (Jackson 1989).

To analyze this subsystem, we need information relating to:-

i. Arrival process:- how customers arrival e.g. singly or in group (batch or bulk arrivals) how the arrival are distributed in time (e.g. what probability distribution of time between . successive arrivals the inter-arrival time distribution (Bhat 2008) whether there is a finite population of customers or (effectively) an infinite number.

The study of queue deal with quantifying the phenomenon of waiting in the line using a representative measures of performance such as average queue length, average waiting time in a queue and average facility utilization (Taha2002), some of the analysis that can be derived using queuing include the expected waiting time in the queue, average waiting time in the system, the expected queue length, expected number of customers served at one time, the probability of balking customers as well as the probability of the system to be in certain state such as empty or full (Patel etal2012)

Queuing models provide the analyst with a power tool for designing and evaluating the performance of queuing system (bank etal2001) any system in while arrival place demand upon a timely capacity resource. May be termed as queuing system.if the arrival time of these demands are unpredictable, then conflicts for the use of resources will raise and queues of waiting customers will form and the length of these queue depend on two aspects of the flow pattern,
First they depend on the average rate, secondly they depend on the statistical fluctuation of this rate (Klenrock 1975).
In 1909, the first study of queuing theory was done by a Danish mathematician AK Erlang which resulted into worldwide acclaimed Erlang telephone models. (Welson 2001).
He examining the telephone network system and tried to determine the effect of fluctuating service demands on cell on utilization of automatic dial equipment. The original problem erlang treated was the calculation of this delay for one telephone operator and in 1917, the results were extended to the activities of several telephone operators (Carson 2001).
That was the same year that erlang publish his well known work "solution of some problem in the theory of probability of significant in automatic telephone exchange"
Development in the field of telephone, traffic continued largely along the lines initiated by erlang and the main publication were those of Molina in 1927 and thoruton D fry 1928.
Many primary care office and other medical practices regularly experience long backlogs for appointments, these backlogs are exacerbated by a significant level of last minutes cancellation or "no shows" which have the effect of wasting capacity. In this paper, we conceptualize such as appointment system as a single server queuing system in which customers who are about to enter services have a state dependent probability of not being served and may rejoin the queue. The derived. Stationary distribution of the queue size, assuming both deterministic as well as exponential services time and compare the performance metrics to the result of a simulation of the appointment system. Our results demonstrate the usefulness of the queuing models in providing guidance on identifying patient Panel size for medical practices that are trying to implement a policy of advance access. (Green and Savin 2008).
In application of queuing theory to automated teller machine (ATM). This paper presents the importance of applying queuing theory to the automated teller machine (ATM) using Monte Carlo simulation in order to determine control and manage the level on queuing congestion found within the automated teller

machine (ATM) centre in Nigeria and also it contain the empirical data analysis of the queuing system obtained at the automated teller machine (ATM) located within bank premises for a period of three months.
Monte Carlo simulation is applied to this study in order to review the queuing discipline at the automated teller machine facilities or automated teller machine service centers and also estimate the arrivals time, waiting time and service of each customer found off peak hours.
An experiment was been carried out with the aid of a stop watch, recording material etc. on order to obtain the time in which every customers spends at automated teller machine (ATM) service centre from the time of arrival to the time of departure. The model contains fie serves which are heavily congested during the peak hours and during the off peak hours servers are found being idle. (Manuel and Offiong 2014).
The aim of this research is to develop an efficient fair queuing model that is capable of reducing congestion by allocating resources on the network between contending users. The proposed model gives higher priority to real time in order to allow them to have dependable performance, Simulation of this propose model is carried out using queuing performance parameters like complexity, through put and delay time of the information our simulation and analysis demonstrate the effectiveness of our proposed model. Its adequately compare with previous fair queuing schemes. (Abikoye etal 2013). An empirical study was conducted to analyzed how waiting in queue in the context of a retail store affects customers purchasing behavior our methodology combine a never data set with periodic information about the queuing system with point of sales data we find that waiting in queue has a non linear impact on purchase incidence and that customers appear to focus mostly on the length of the queue, without adjusting enough for the speed at which the moves. An implication of this finding is that pooling multiple queues into a single queue may increase the length o the queue into a single observed y customer and thereby lead to lower revenues. We also find that customers sensitivity to waiting is heterogonous and negatively correlation with price sensitivity, which has important implications for pricing in a muiti-product category subject to congestion effects. (Yina etal 2011). The data collection being used in this study is primarily collected by direct interview and direct observation of the withdrawal section of the banking hall using a wristwatch. The main aim of this research is to show how the management of first bank will go about the reduction of the waiting time of customers.

This piece of work will also check if increasing the number of server will reduce the waiting time as well as putting the profit in consideration.
Hence, the objective of this project will be achieved by analyzing the real life observed data, them constructing a new model of system and using statistical analytical tools like Poisson, exponential and chi square distribution to study pattern and reaction to change in the system.

Area of Study
This research centers on the waiting area of the banking hall of first bankbauchi main branchi.e the withdrawal section of the bank.

Method of Data Collection
First, we use the chi square distribution to study the pattern and reactions to change of the system. i.e. identification of the customers, serve and queue characteristics that are apparent in the system, after which we analyze the data collected from the bank based on

I. The arrival time of each customer
II. The time service commenced for each customer leaves the system using the m/m/s model since the system has to do with multiple server and a single queue.

1M/M/S System
Is a queuing process having a Poisson distribution inter-arrival time, S servers, with S dependent identically distributed exponential service time (which do not depend on the state of the system infinite capacity and a first com first serve queue discipline
The arrival pattern being state independent $\lambda n = \lambda$ for all n, the service time associates with each server are also state independent, but since the number of server that actually attend to customers (i.e. are not idle) does not depend on the number of customers in the system, the effective time it take the system to process customers through the service facility is state independent.
In particular, If $= \lambda$ is men services time for one server to handle one customer then the mean rate of service completion when they are n customs in the system is given by
$\mu n =$
The steady state condition prevail whenever $=< 1$
The steady state probability is given by
P_n

Where
P_O^{-1} =
Lq=
Now from Wq =, Wq =
Ws = Wq+=
Ls = λWs =
WL(t) = where (t ≥ 0)
Wq(t) = where (t ≥ 0)

Method of data collection

The data was collected primarily by direct observation at the bank, thus the research recorded the following event as it happened in the system using a wrist watch.

i. The arrival time of each customers
ii. The time service commerce for each customers in the system
iii. The time customer leave the system

These events were observed at the withdrawal section of the banking hall. One week of two working days was spent to collect relevant data.

Data Presentation

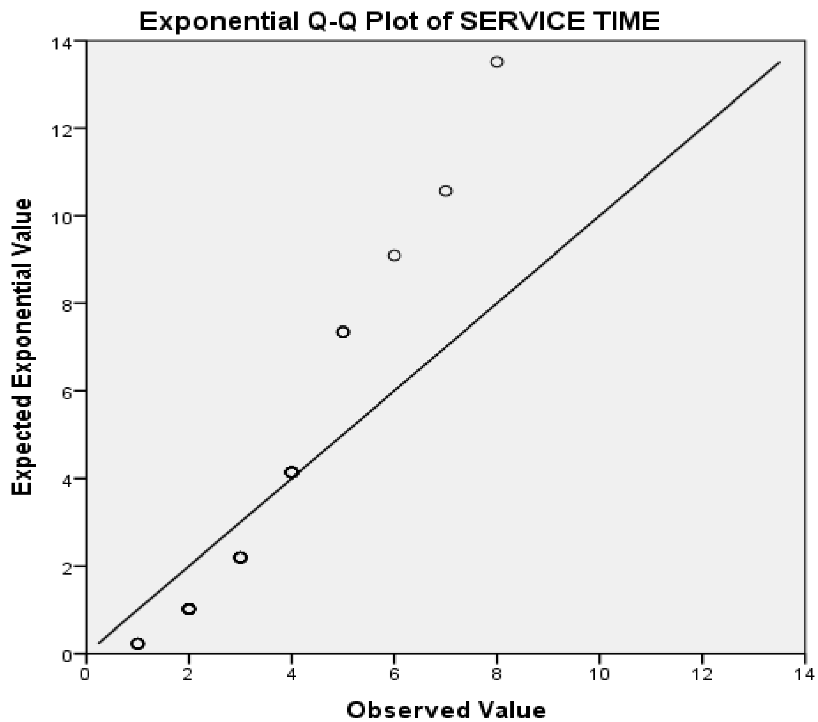
First bank was considered and the methodology was through observation and the data were captured on the field to enthrone originality of the work.
The observation concerned itself with the arrival time of customers and the time in which they were served by the system. The researcher also observed the queuing discipline and also the patience being exercised by customers in order to get served by the system.
The queuing system was a multi- channel multi-server queuing and service system, the researcher, through observation found out four servers and queues waiting for service delivery. The observation was done for two days and the analysis was done using advanced Microsoft excel for arithmetic computations and SPSS packages for most complex analysis. Prior to the analysis being done, data were properly check, cleansed and coded using SPSS software. It should be bore in mind that the arrival time was a Poisson distribution. Using a cross tabulation, This was as confirmed by the researcher after testing the actual arrival and the expected arrival and the service time was assume to be exponentially distributed after doing same.

Tabe1:
Model Information

Dependent Variable	ARRIVAL TIME
Probability Distribution	Poisson
Link Function	Log

SOURCE SPSS OUTPUT

The above was a confirmatory table the arrival rate follows a poisson distribution



Above is the graph showing the exponential distribution of the service time.

The arrival time is given as λ, while the service mean time is given as μ. The inter-arrival time and the mean time is and respectively. Other formulas relevant to the research questions and subsequent analysis were carried out by the researcher.

From the table above, the customers observed were 50 in day one and the earliest arrival as observed from the table as captured by the researcher is 8.15am and the last arrival from the table was 10.45am. The difference between the time is given as from 8.15 to 10.45 is 138 minutes representing 2 hours 18minutes. If we divide the number of customers and the time of arrival, we will get the average number of customers arriving the system per minutes.

λ= number of customers/ time

= 50/138 = 0.36

μ= sum of service time/ customers

=138/50 = 2.76

1/ λ=2.77 this is the inter-arrival rate of customers to the banking hall which follows a Poisson distribution

S=4

When $\mu s > \lambda$ it shows that the service capacity of the system exceeds customers requirement. This is also the case when T_I is less than one ($T_I < 1$)

For the day two observation.

μ = 2.54 λ=0.39

= 0.4

= 2.8 the traffic intensity factor is

We are interested in the overall data average form the foregoing table, there were 50 customers observed and four different channels. The early arrival time to the least time observed in the table shows that between (8.15am -10.45am) is a period of 138minutes. From the customers observed, they were 50 in number that arrived at different interval and joined the queue in first bank branch. Hence the system under consideration

The mean arrival rate is given as λ = number of customers/time duration

λ =50/138 = 0.36 customers/minutes

This implies that the customers arrive at the bank at an average of approximately 0.3 per minute, which is that λ is equal to 0.3 that are the average numbers of customers that joined the queue in a minute. Though there were four channels and the customers evenly distributed themselves across the servers.

S is given as the available numbers of channels or servers in the banking hall, and this was S as observed by the researcher as captured on the extreme left of the table above.

Therefore S = 4

Traffic intensity or utilization factor. This shows the level of how the servers are busy

The traffic intensity factor is given as

= 0.36/4 * 3.08

. that of the day two observation was 0.038.

Therefore this implies that the traffic intensity factor is very low and the servers are very efficient in delivery services in record time. Again this also buttresses the fact that

1. To what extent does service time differ from the actual time that customers have to wait before being served?

The mean or expected waiting time for the customers in the queue is given as the average time spent waiting in line is given as

= 5.93/ 0.36

. Same calculation was applied on the day two observations and the answer was 15.6 which is also approximately 16 minutes as the first days observation.

The average number of customers waiting in line to be served by the system is given as?

This implies that on average approximately 6 customers are waiting on queue to be served.

The number of customers divided by 6 and the average time taking to be served on the queue is given as 16/6= 2.6 minutes.

The extent to which service time differ from the actual time the customer has to wait before being served is gotten from the computations of the service time per customer and the time frame the customer has to wait in the queue before being served.

The mean service time is given as μ which is 3.08 and the actual time a customer has to wait before being served is given as w_q= 16-3.08= 13minutes. It should however be bore in mind that the researcher output was the averages.

1. To what extent does poor service facilities pattern affect queue discipline?

As for the efficiency of the channel or the servers, from the foregoing the system was efficient and the servers were delivering in record time. Systems inefficiency resulting in Queue indiscipline such as balking (i.e. customers refusing to join queue because it’s too long), jockey (switching between queues) or line cutting or shortening was not observed. This may be attributed to the fact that system was not overloaded and has limited and efficient service time. Poor service facility pattern no doubt affects queue discipline but in this case it was not observed. In the event of poor service facility, queue indiscipline may arrive resulting in balking, reneging, jockeying and shortening. Service facility was okay in this observation as seen in the traffic intensity factor of 0.1 which is very low.

2. To what extent does average service time affect the overall performance of the bank?

1/ μ=0.36 this is the average time rate of services delivered to a customer to the banking hall which follows an exponential distribution

The average service time is given as μ which is 3.08 this shows that on the average the time spent by the customers to have service being delivered is on average approximately 3minutes. From the observation of the figures, it shows that the system requires 3minutes to have services delivered to the waiting customers.

The overall performance of the servers was very okay in that 3minutes is not large a time frame to reduce the performance of the channels. Though some of the customers were served less than the time in view while others were more than this time. In the event of where the overall performance of the bank is affected due to the time frame of the service can lead to queue indiscipline as earlier cited above. That is reneging, balking, jockeying etc. form comparing the 2 days observation, there was no much difference and this is attributed to the consistency and efficiency of the system.

3. To what extent does service facilities affect customer service

   The average utilization of the system is given by the formula

$= {}^{-1}$

0.837 Approximately 0.84. This shows that the probability that the system will become idle is 0.84 which is 84%. The calculation for day two observation revealed 87%

Customers were spontaneously served by the four servers and the tendency to have a long queue was not there. This is as revealed by the queue traffic intensity factor of 0.024 which is very low, the traffic intensity factor for day 2 showed 0.038 which is lower for day 2 observation. This had no much discrepancy between day 1 and 2 of the observation.

4. How does poor service delivery affect customer's behavior?

As earlier articulated customers may not be patient to wait in the event of a very high traffic and this may lead to committing unethical behavioral standard. The more there are channels or servers, the more efficient the system become.

   a. It leads to jockeying
   b. Balking
   c. Reneging
   d. Shunting

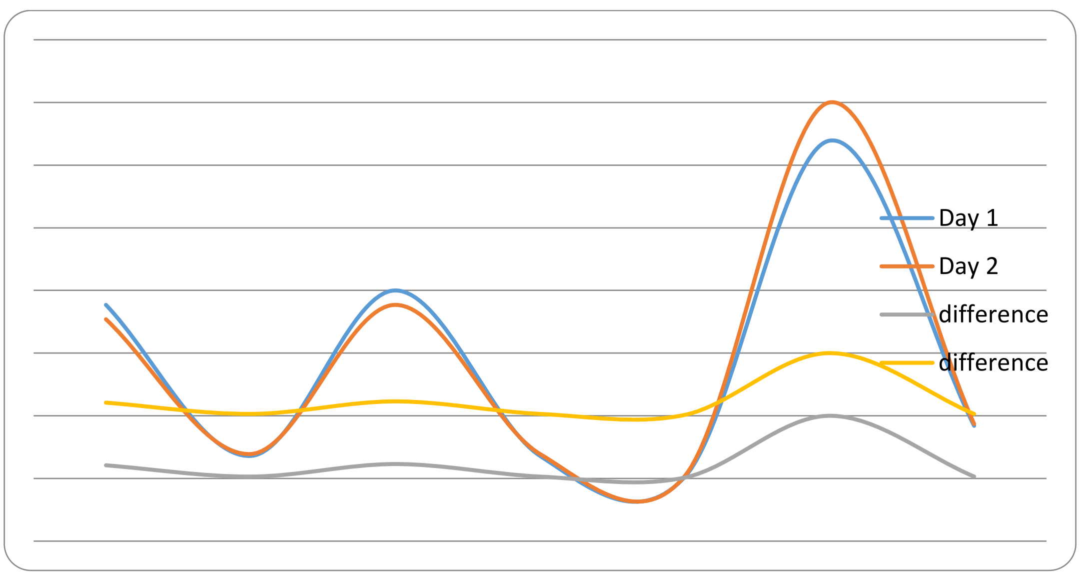
Poor service delivery emanates from the situation where the arrival rate is very high and the service rate is very low. This can bring about long queue which will eventuate the above listed queue indiscipline.

When λ is less than μ there you have systems idleness or where you have T_I close to zero you have a large systems service capacity delivery. When this becomes the case, customers tend to do otherwise.

Table 2:

S/N		Day 1	Day 2	Differences
1	Arrival rate	2.77	2.54	0.21
2	Service rate	0.36	0.39	0.03
3	Inter - arrival rate	3.0	2.77	0.23
4	Average service time	0.36	0.39	0.03
5	Traffic intensity factor	0.024	0.038	0.014
6	Average Number of those waiting in the queue to be served	5.39	6	1
8	The probability that the system is idle	0.84	0.87	0.03
9	Average time spent by customers waiting in line	16	15.6	0.4

From the foregoing tables, it shows that the difference in terms of the queuing indicators for day one and day 2 observations did not show much discrepancy to enable the researcher pass a judgment on the findings. The inter-arrival rate and the service rate were similar apart from the utilization factor.



The formula applied are given below

Interarrival rate

Average service rate

The probability that the system is idle

The traffic intensity factor

The systems utilization factor

The mean expected time

Summary

From the table below, there are **50** observed customers both in day one and day two starting from **8.15 – 10. 45** by four servers representing**2** hour**18**minutes. Considering the analytical solution, the capacity of the system under study is about **100** customers and the arrival rate is **5.31** while the service rate is **0.75.** This shows that the arrival rate in the system is greater than the service rate and this implies that customers have to queue up though the queue will not be long. The probability that servers are idle is **0.84** for day one and **0.87** for day two which shows that the servers will be **1.71%** and **0.03%** busy for both day one and two.

The average number of waiting in the queue is **5.93** for day one and **6** for day two from the foregoing queue measures the average time spent by customer waiting in the line is **16** and **15**respectively.

Conclusion

At the end of this research work, the researcher now concluded that adding more servers will help reduce the time customer spent in the queue and as well to reduce cost incurred from waiting, hence the objective of this research is achieved.

The advantage of using this multiple queuing in parallel in the system with multiple servers also in parallel is that a slow server does not affect the movement of the queue i.e. if a server is slow it does not affect the movement of the queue because next customer can go to the next available server instead of waiting for the slow server.

Recommendation

We have recommended that the management of First Bank Bauchi Branch to increase the number of servers at the withdrawal section as well as to increase in their I.C.T application so as to reduce the time customer has to wait to receive services, thereby minimizing the cost incurred from waiting.

Source of the table is from First Bank Bauchi Branch
DAY ONE

S/N	SERVERS	ARRIVAL TIME	SERVICE BEGIN TIME (a.m)	SERVICE ENDS TIME (a.m)	SERVICE DELIVERY TIME (a.m)
1	4	8.15	8.17	8.21	4
2	3	8.18	8.19	8.23	4
3	4	8.21	8.24	8.25	1
4	3	8.23	8.25	8.31	6
5	2	8.24	8.28	8.32	4
6	2	8.27	8.31	8.34	3
7	3	8.29	8.34	8.38	4
8	2	8.31	8.37	8.41	4
9	2	8.34	8.41	8.44	3
10	3	8.36	8.43	8.45	2
11	1	8.38	8.45	8.47	2
12	1	8.41	8.47	8.48	1
13	2	8.43	8.51	8.53	2
14	3	8.45	8.55	8.58	3
15	1	8.47	8.59	9.02	3
16	2	8.49	9.01	9.05	4
17	3	8.52	9.02	9.07	5
18	1	8.54	9.05	9.07	2
19	2	8.56	9.07	9.09	2
20	3	8.59	9.09	9.11	2
21	4	9.02	9.11	9.15	4
22	3	9.07	9.15	9.18	3
23	2	9.08	9.18	9.22	4
24	3	9.11	9.21	9.23	2
25	2	9.14	9.22	9.26	4
26	3	9.18	9.24	9.27	3
27	1	9.21	9.27	9.28	1
28	1	9.26	9.31	9.33	2
29	4	9.31	9.36	9.38	2
30	3	9.36	9.37	9.38	1
31	1	9.42	9.44	9.47	3

32	2	9.47	9.51	9.55	4
33	3	9.49	9.52	9.54	2
34	2	9.52	9.53	9.57	4
35	3	9.58	10.01	10.05	4
36	4	10.02	10.04	10.05	1
37	4	10.04	10.07	10.11	4
38	2	10.09	10.11	10.14	3
39	1	10.12	10.15	10.16	1
40	2	10.15	10.17	10.19	2
41	3	10.18	10.22	10.27	5
42	2	10.21	10.26	10.31	5
43	4	10.24	10.31	10.39	8
44	2	10.27	10.35	10.37	2
45	3	10.31	10.39	10.41	2
46	4	10.33	10.41	10.44	3
47	3	10.36	10.41	10.42	1
48	3	10.39	10.42	10.46	4
49	4	10.42	10.44	10.46	2
50	1	10.45	10.51	10.58	7

DAY TWO

S/ N	SERVERS	ARRIVAL TIME	SERVICE BEGIN	SERVICE ENDS	SERVICE TIME	SERVICE TIME
1	3	8.15	8.21	8.25	0.04	4
2	2	8.16	8.17	8.21	0.04	4
3	1	8.19	8.21	8.22	0.01	1
4	4	8.21	8.22	8.25	0.03	3
5	1	8.23	8.24	8.26	0.02	2
6	4	8.25	8.27	8.31	0.04	4
7	2	8.27	8.28	8.31	0.03	3
8	2	8.29	8.31	8.33	0.02	2
9	3	8.31	8.33	8.36	0.03	3
10	1	8.33	8.34	8.38	0.04	4
11	2	8.35	8.36	8.37	0.01	1
12	3	8.37	8.38	8.41	0.03	3
13	3	8.39	8.41	8.45	0.04	4
14	3	8.41	8.44	8.55	0.11	11

15	4	8.43	8.45	8.47	0.02	2
16	4	8.46	8.47	8.51	0.04	4
17	4	8.48	8.51	8.53	0.02	2
18	1	8.51	8.53	8.56	0.03	3
19	1	8.52	8.55	8.58	0.03	3
20	2	8.54	8.56	8.57	0.01	1
21	3	8.56	8.58	9.02	0.04	4
22	2	8.58	9.01	9.02	0.01	1
23	1	9.01	9.03	9.06	0.03	3
24	2	9.04	9.04	9.07	0.03	3
25	3	9.06	9.08	9.09	0.01	1
26	2	9.11	9.12	9.13	0.01	1
27	4	9.14	9.15	9.16	0.01	1
28	3	9.16	9.18	9.21	0.03	3
29	2	9.21	9.22	9.25	0.03	3
30	3	9.23	9.23	9.26	0.03	3
31	2	9.26	9.27	9.31	0.04	4
32	1	9.29	9.29	9.32	0.03	3
33	2	9.32	9.33	9.34	0.01	1
34	3	9.36	9.37	9.39	0.02	2
35	3	9.39	9.41	9.42	0.01	1
36	3	9.42	9.43	9.44	0.01	1
37	2	9.45	9.46	9.47	0.01	1
38	3	9.48	9.51	9.53	0.02	2
39	3	9.52	9.53	9.56	0.03	3
40	1	9.55	9.55	9.57	0.02	2
41	2	10.01	10.02	10.04	0.02	2
42	1	10.05	10.06	10.08	0.02	2
43	3	10.11	10.11	10.15	0.04	4
44	2	10.15	10.15	10.16	0.01	1
45	4	10.18	10.21	10.22	0.01	1
46	4	10.22	10.24	10.25	0.01	1
47	3	10.25	10.26	10.28	0.02	2
48	1	10.31	10.32	10.33	0.01	1
49	2	10.36	10.37	10.39	0.02	2
50	1	10.45	10.48	10.52	0.04	4

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